

# **UNIVERSITY OF NAIROBI**

**DEPARTMENT OF ELECTRICAL AND ELECTRONICS  
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**AUTHOR : SEBASTIAN M. MUTHUSI**

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**SUPERVISOR : DR. MANG'OLI**

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**PROJECT TITLE : MODELING WARD LEONARD  
SPEED CONTROL SYSTEM**

**SUBJECT:**

**TO TAKE THE NECESSARY MEASUREMENT AND TO  
DESIGN AND CONSTRUCT A THREE KILOWATT SET.**

**THIS PROJECT IS SUBMITTED IN PARTIAL  
FULFILMENT FOR THE AWARD OF THE DEGREE OF  
BSc. ELECTRICAL & ELECTRONIC ENGINEERING,  
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## ABSTRACT

A D.C. Generator is connected in series opposed to the polarity of a D.C. power source supplying a dc. Drive motor. The back ground of the ward Leonard speed control system is explained in chapter two. It was explained by use of different theories for instance the use of proportional integral controller and why it was preferred to proportional integral derivatives (PID) and the proportional derivative (PD). The integral and the derivative part are also explained in this chapter. Different constants involved have also been considered including the motor constants, generator constant and torque constant among others. To obtain these constants data is collected from an industry and different graphs are plotted and their gradients calculated which are considered to be the required constants. By use of these constants the modeled equations in chapter three are solved. stability of the DC motor is determined by solving the Routh's Hurwitz criterion. Also for stability the open loop and closed loop transfer functions are simulated using matlab and the nyquist, bode plot, root locus and the Nichols chart are plotted. The system appears to be very stable and having considerable steady state error and therefore concluded that the objectives were realized

The aim of the project was to design and construct an electronic speed control unit capable of operating the closed loop ward Leonard speed control system in the machines laboratory.

Unfortunately the ward Leonard speed control system in the machines lab was faulty ;that is the load had short circuited since it has been rained on. Efforts to change the load did not succeed because its resistor bank had very unique ratings from any other load in the machines lab. This work hence was carried out at the Kenya airports authority for the determination of the relevant constants in the main substation and substation no 4.

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# CHAPTER ONE

## INTRODUCTION

In many industrial applications it is important to be able to make accurately crawling speed as well as high speed.

The Ward Leonard speed control system is well suited for this. Invented by American investor whose name it bears, this system consists essentially of a prime-mover, a variable voltage dc generator and a dc work motor

The prime mover may consist of a steam turbine, a water turbine or as is usually the case of a three phase induction motor

Although an adjustable voltage installation of this type is rather expensive, involving as it does three machines to control a motor it does nevertheless find wide application wherever low and high speeds must be accurately made and where the service is severe and exacting such as in mill areas, and where dc motors are still preferred to ac motors due to the requirement of the superior performance characteristics of the machines

In addition the Ward Leonard speed control system offers the following advantages:-

- The motor is started, accelerated; speed adjusted and stopped by the mere adjustment of a single potential meter which in turn adjusts the Ward Leonard generator voltage.
- Many generators having special characteristics can be employed to match specific motor load requirements. This is particularly desirable in certain machine tools for such heavy equipment as excavators
- Magnetic and rotating amplifiers with their outstanding and astonishing performance characteristics can be readily adapted to this system of control.
- Where this is done control power can be greatly reduced and regulation is considerably improved.

Chapter two is a clear explanation of theories used, diagrams tables advantages and disadvantages of choosing one criterion to the other. This is also the general overview of the whole project report.

To be able to design such a control unit one has to be familiar with the system as well as know the relevant constants involved. To satisfy the former requirement an analysis of the general closed loop system is carried out in chapter three. Here is also the modeling control equation and assumptions are made to approximate the system with a second order system and the stability of the general system is examined. In chapter 3 also the second demand is satisfied by making measurements on a particular system . The general system is linked with the particular system for which the speed control unit is to be designed.

The measurements are further employed in predicting the system dynamic and in stability studies. By application of the root locus,nyquist plot, bode plot and the Nichols chart techniques we confirm what had been predicted by the Routh Hurwitz criterion in this chapter.

Chapter 4 is overview look at the preliminary results summarize and try to solve them. The equations modeled in chapter three are solved using the constants determined from the graphs in that chapter.

Chapter five is the conclusion of the including the comparison of practical values with theoretical values.

## CHAPTER TWO

### 2.0 DC GENERATORS

When used as a power amplifier a DC generator would have a power rating to match that of the load and would be driven at nominally Constant speed. Usually by a three phase induction motor of comparable power rating as shown in fig 1

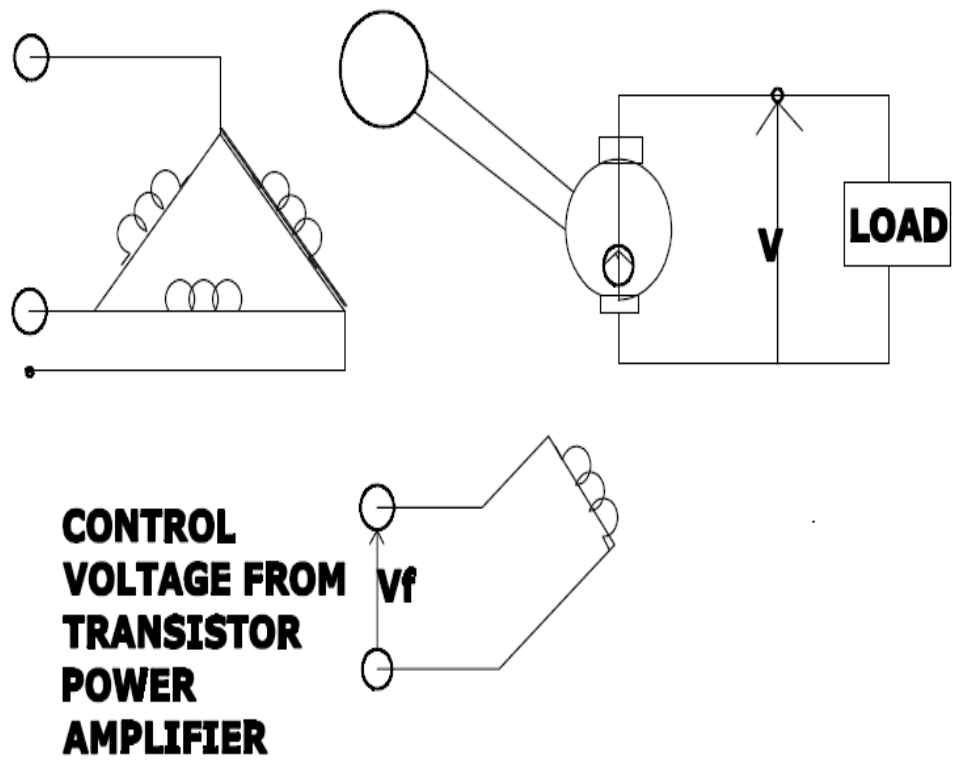
Typically the power gain might be in the order of 1000 where as the voltage gain would be in the range of 1 to 10 the advantage of draining a good current wave form from the AC main must be offset against the capital cost of two machines to match the load in terms of two machines to match the load in terms of power rating, together with the reliability and maintainability limitations of commutator machine where the generator output drives the armature of a comparably rated DC drive motor, the system becomes a Ward Leonard set the generator field sometimes may be center tapped or split, to suit the requirements of the transistor power amplifier . Regenerative braking is inherent, where by the motor can be accelerated by returning energy via the other two machines, back into the AC machines The Ward Leonard motor control system was Mr. Leonard's best known and most lasting invention.



## 2.1 HOW IT WORKS

Leonard had patents for more than 100 inventions during his lifetime, but is best known for the Ward Leonard motor control system. The Ward Leonard system was devised in 1891. It involves a prime mover (usually an AC or alternating current motor) which operates a direct current or DC generator at a consistent speed. The framework of the generator is connected to a direct current or DC motor. The motor in turn, is responsible for adjusting the speed of the equipment and does so by altering the output voltage with the generator with the help of a rheostat. The flow of motor field typically stays unaltered and can be reduced at times to increase the speed of the base. Ward Leonard systems typically include an exciter generator that is operated by the prime mover in order to field power supply from the DC exciter.

The Ward Leonard motor control system was Mr. Leonard's best known and most lasting invention. In Ward Leonard system, a prime mover drives a direct current (DC) generator at a constant speed. The armature of the DC generator is connected directly to the armature of a DC motor, the DC motor drives the load equipment at an adjustable speed. The motor speed is adjusted by adjusting the output voltage of the generator using a rheostat to adjust the excitation current in the field winding. The motor field current is usually not adjusted but the motor field is sometimes reduced to increase the speed above the base speed. The prime mover is usually an alternating current (AC) motor, but a DC motor or an engine might be used instead. To provide the DC field excitation power supply, Ward Leonard systems usually include an exciter generator that is driven by the prime mover.



**FIG 1:THREE PHASE INDUCTION MOTOR**

Itemref	Quantity	Title/Name, designation, material, dimension etc			Article No./Reference	
Designed by sebastian muthusi	Checked by jeremiah	Approved by - date 20/05/2009	File name ward leonard dia	Date 20/05/2009	Scale 1:1	
Dr mang'oli			introduction diagram			
			number one		Edition 2009	Sheet 1/1

## 2.1 PID CONTROLLER

A **proportional–integral–derivative controller (PID controller)** is a generic control loop feedback mechanism (controller) widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set point by calculating and then outputting a corrective action that can adjust the process accordingly and rapidly, to keep the error minimal.

### 2.1.1 General schematic diagram

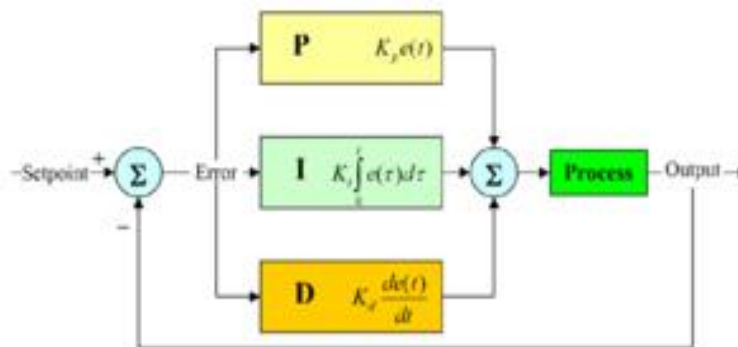


Fig 2.1

A block diagram of a PID controller

The PID controller calculation involves three separate parameters; the proportional, the integral and derivative values. The *proportional* value determines the reaction to the current error, the *integral* value determines the reaction based on the sum of recent errors, and the *derivative* value determines the reaction based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element such as the position of a control valve or the power supply of a heating element.

By tuning the three constants in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point and the degree of system oscillation. Note

that the use of the PID algorithm for control does not guarantee optimal control of the system or system stability.

Some applications may require using only one or two modes to provide the appropriate system control. This is achieved by setting the gain of undesired control outputs to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are particularly common, since derivative action is very sensitive to measurement noise, and the absence of an integral value may prevent the system from reaching its target value due to the control action.

Note: Due to the diversity of the field of control theory and application, many naming conventions for the relevant variables are in common use.

## **2.2 CONTROL LOOP BASICS**

A familiar example of a control loop is the action taken to keep one's shower water at the ideal temperature, which typically involves the mixing of two process streams, cold and hot water. The person feels the water to estimate its temperature. Based on this measurement they perform a control action: use the cold water tap to adjust the process. The person would repeat this input-output control loop, adjusting the hot water flow until the process temperature stabilized at the desired value.

Feeling the water temperature is taking a measurement of the process value or process variable (PV). The desired temperature is called the set point (SP). The output from the controller and input to the process (the tap position) is called the manipulated variable (MV). The difference between the measurement and the set point is the error (e), too hot or too cold and by how much.

As a controller, one decides roughly how much to change the tap position (MV) after one determines the temperature (PV), and therefore the error. This first estimate is the equivalent of the proportional action of a PID controller. The integral action of a PID controller can be thought of as gradually adjusting the temperature when it is almost right. Derivative action can be thought of as noticing the water temperature is getting

hotter or colder, and how fast, anticipating further change and tempering adjustments for a soft landing at the desired temperature (SP).

Making a change that is too large when the error is small is equivalent to a high gain controller and will lead to overshoot. If the controller were to repeatedly make changes that were too large and repeatedly overshoot the target, this control loop would be termed unstable and the output would oscillate around the set point in either a constant, growing, or decaying sinusoid. A human would not do this because we are adaptive controllers, learning from the process history, but PID controllers do not have the ability to learn and must be set up correctly. Selecting the correct gains for effective control is known as tuning the controller.

If a controller starts from a stable state at zero error ( $PV = SP$ ), then further changes by the controller will be in response to changes in other measured or unmeasured inputs to the process that impact on the process, and hence on the PV. Variables that impact on the process other than the MV are known as disturbances. Generally controllers are used to reject disturbances and/or implement set point changes. Changes in feed water temperature constitute a disturbance to the shower process.

In theory, a controller can be used to control any process which has a measurable output (PV), a known ideal value for that output (SP) and an input to the process (MV) that will affect the relevant PV. Controllers are used in industry to regulate temperature, pressure, flow rate, chemical composition, speed and practically every other variable for which a measurement exists. Automobile cruise control is an example of a process which utilizes automated control.

Due to their long history, simplicity, well grounded theory and simple setup and maintenance requirements, PID controllers are the controllers of choice for many of these applications.

## 2.3 PID CONTROLLER THEORY

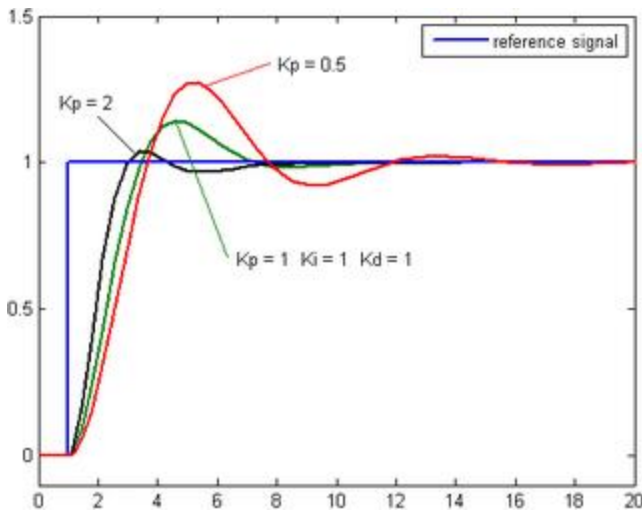
This section describes the parallel or non-interacting form of the PID controller. For other forms please see the Section "Alternative notation and PID forms".

The PID control scheme is named after its three correcting terms, whose sum constitutes the manipulated variable (MV). Hence:

$$MV(t) = P_{out} + I_{out} + D_{out} \tag{2.1}$$

where  $P_{out}$ ,  $I_{out}$ , and  $D_{out}$  are the contributions to the output from the PID controller from each of the three terms, as defined below.

### 2.3.1 Proportional term



**Fig 2.2**

Plot of PV vs time, for three values of  $K_p$  ( $K_i$  and  $K_d$  held constant)

The proportional term (sometimes called *gain*) makes a change to the output that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant  $K_p$ , called the proportional gain.

The proportional term is given by:

$$P_{out} = K_p e(t) \tag{2.2}$$

Where

- $P_{out}$ : Proportional term of output
- $K_p$ : Proportional gain, a tuning parameter
- $e$ : Error =  $SP - PV$
- $t$ : Time or instantaneous time (the present)

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable (See the section on loop tuning). In contrast, a small gain results in a small output response to a large input error, and a less responsive (or sensitive) controller. If the proportional gain is too low, the control action may be too small when responding to system disturbances.

In the absence of disturbances, pure proportional control will not settle at its target value, but will retain a steady state error that is a function of the proportional gain and the process gain. Despite the steady-state offset, both tuning theory and industrial practice indicate that it is the proportional term that should contribute the bulk of the output change.

### 2.3.2 Integral term

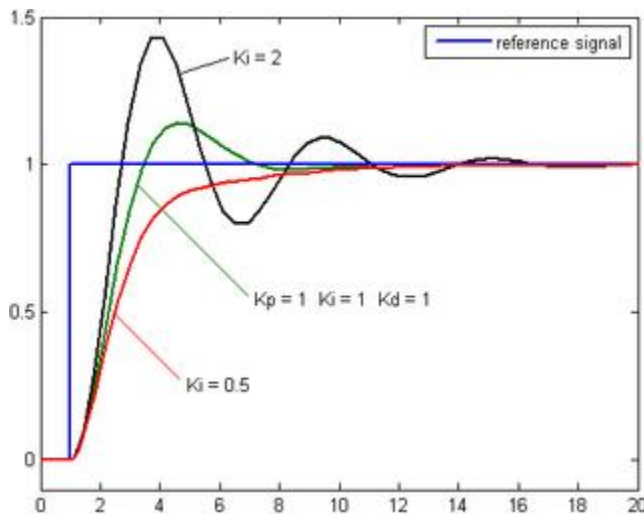


Fig 2.3

Plot of PV vs time, for three values of  $K_i$  ( $K_p$  and  $K_d$  held constant)

The contribution from the integral term (sometimes called *reset*) is proportional to both the magnitude of the error and the duration of the error. Summing the instantaneous error over time (integrating the error) gives the accumulated offset that should have been corrected previously. The accumulated error is then multiplied by the integral gain and added to the controller output. The magnitude of the contribution of the integral term to the overall control action is determined by the integral gain,  $K_i$ .

The integral term is given by:

$$I_{\text{out}} = K_i \int_0^t e(\tau) d\tau \quad \text{..2.3}$$

Where

- $I_{\text{out}}$ : Integral term of output
- $K_i$ : Integral gain, a tuning parameter
- $e$ : Error =  $SP - PV$
- $t$ : Time or instantaneous time (the present)
- $\tau$ : A dummy integration variable

The integral term (when added to the proportional term) accelerates the movement of the process towards set point and eliminates the residual steady-state error that occurs with a proportional only controller. However, since the integral term is responding to accumulated errors from the past, it can cause the present value to overshoot the set point value (cross over the set point and then create a deviation in the other direction). For further notes regarding integral gain tuning and controller stability, see the section on loop tuning.



### 2.3.3 Derivative term

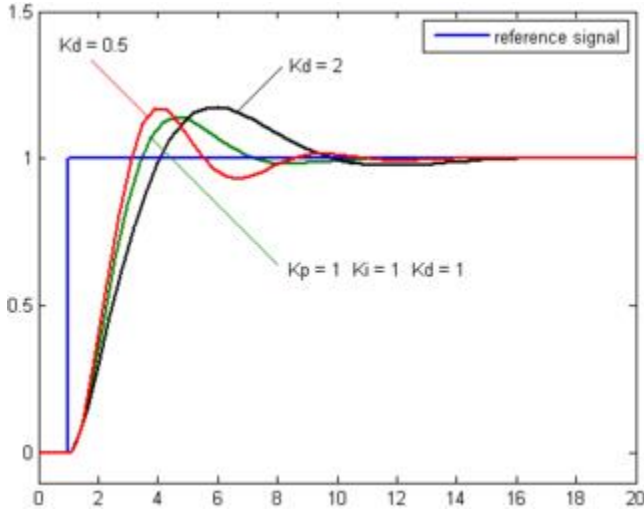


Fig 2.4

Plot of PV vs time, for three values of  $K_d$  ( $K_p$  and  $K_i$  held constant)

The rate of change of the process error is calculated by determining the slope of the error over time (i.e., its first derivative with respect to time) and multiplying this rate of change by the derivative gain  $K_d$ . The magnitude of the contribution of the derivative term (sometimes called *rate*) to the overall control action is termed the derivative gain,  $K_d$ .

The derivative term is given by:

$$D_{out} = K_d \frac{de}{dt}(t) \quad \dots 2.4$$

Where

- $D_{out}$ : Derivative term of output
- $K_d$ : Derivative gain, a tuning parameter
- $e$ : Error =  $SP - PV$
- $t$ : Time or instantaneous time (the present)

The derivative term slows the rate of change of the controller output and this effect is most noticeable close to the controller set point. Hence, derivative control is used to

reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability. However, differentiation of a signal amplifies noise and thus this term in the controller is highly sensitive to noise in the error term, and can cause a process to become unstable if the noise and the derivative gain are sufficiently large.

## 2.4 SUMMARY

The proportional, integral, and derivative terms are summed to calculate the output of the PID controller. Defining  $u(t)$  as the controller output, the final form of the PID algorithm is:

$$u(t) = MV(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt}(t)$$

and the tuning parameters are:

Proportional gain,  $K_p$

larger values typically mean faster response since the larger the error, the larger the Proportional term compensation. An excessively large proportional gain will lead to process instability and oscillation.

Integral gain,  $K_i$

larger values imply steady state errors are eliminated more quickly. The trade-off is larger overshoot: any negative error integrated during transient response must be integrated away by positive error before we reach steady state.

Derivative gain,  $K_d$

larger values decrease overshoot, but slows down transient response and may lead to instability due to signal noise amplification in the differentiation of the error.

## 2.5 MANUAL TUNING

If the system must remain online, one tuning method is to first set  $K_i$  and  $K_d$  values to zero. Increase the  $K_p$  until the output of the loop oscillates, then the  $K_p$  should be left set to be approximately half of that value for a "quarter amplitude decay" type response.

Then increase  $K_i$  until any offset is correct in sufficient time for the process. However, too much  $K_i$  will cause instability. Finally, increase  $K_d$ , if required, until the loop is acceptably quick to reach its reference after a load disturbance. However, too much  $K_d$  will cause excessive response and overshoot. A fast PID loop tuning usually overshoots slightly to reach the set point more quickly; however, some systems cannot accept overshoot, in which case an "over-damped" closed-loop system is required, which will require a  $K_p$  setting significantly less than half that of the  $K_p$  setting causing oscillation

Effects of <i>increasing</i> parameters				
Parameter	Rise time	Overshoot	Settling time	Error at equilibrium
$K_p$	Decrease	Increase	Small change	Decrease
$K_i$	Decrease	Increase	Increase	Eliminate
$K_d$	Indefinite (small decrease or increase)	Decrease	Decrease	None

**Table 2.1**

## 2.6 ZIEGLER–NICHOLS METHOD

Another tuning method is formally known as the Ziegler–Nichols method, introduced by John G. Ziegler and Nathaniel B. Nichols. As in the method above, the  $K_i$  and  $K_d$  gains are first set to zero. The  $P$  gain is increased until it reaches the critical gain,  $K_c$ , at which the output of the loop starts to oscillate.  $K_c$  and the oscillation period  $P_c$  are used to set the gains as shown:

Ziegler–Nichols method			
Control Type	$K_p$	$K_i$	$K_d$
$P$	$0.50K_c$	-	-
$PI$	$0.45K_c$	$1.2K_p / P_c$	-
$PID$	$0.60K_c$	$2K_p / P_c$	$K_p P_c / 8$

Table 2.2

## 2.7 MODIFICATIONS TO THE PID ALGORITHM

The basic PID algorithm presents some challenges in control applications that have been addressed by minor modifications to the PID form.

One common problem resulting from the ideal PID implementations is integral windup. This problem can be addressed by:

- Initializing the controller integral to a desired value
- Increasing the set point in a suitable ramp
- Disabling the integral function until the PV has entered the controllable region
- Limiting the time period over which the integral error is calculated
- Preventing the integral term from accumulating above or below pre-determined bounds

Many PID loops control a mechanical device (for example, a valve). Mechanical maintenance can be a major cost and wear leads to control degradation in the form of either stiction or a deadband in the mechanical response to an input signal. The rate of mechanical wear is mainly a function of how often a device is activated to make a change. Where wear is a significant concern, the PID loop may have an output deadband to reduce the frequency of activation of the output (valve). This is accomplished by modifying the controller to hold its output steady if the change would be small (within the defined deadband range). The calculated output must leave the deadband before the actual output will change.

The proportional and derivative terms can produce excessive movement in the output when a system is subjected to an instantaneous step increase in the error, such as a large set point change. In the case of the derivative term, this is due to taking the derivative of the error, which is very large in the case of an instantaneous step change. As a result, some PID algorithms incorporate the following modifications:

#### Derivative of output

In this case the PID controller measures the derivative of the output quantity, rather than the derivative of the error. The output is always continuous (i.e., never has a step change). For this to be effective, the derivative of the output must have the same sign as the derivative of the error.

#### Set point ramping

In this modification, the set point is gradually moved from its old value to a newly specified value using a linear or first order differential ramp function. This avoids the discontinuity present in a simple step change.

#### Set point weighting

Set point weighting uses different multipliers for the error depending on which element of the controller it is used in. The error in the integral term must be the true control error to avoid steady-state control errors. This affects the controller's set point response. These parameters do not affect the response to load disturbances and measurement noise.

## 2.8 LIMITATIONS OF PID CONTROL

While PID controllers are applicable to many control problems, they can perform poorly in some applications.

PID controllers, when used alone, can give poor performance when the PID loop gains must be reduced so that the control system does not overshoot, oscillate or *hunt* about the control set point value. The control system performance can be improved by combining the feedback (or closed-loop) control of a PID controller with feed-forward (or open-loop) control. Knowledge about the system (such as the desired acceleration and inertia) can be fed forward and combined with the PID output to improve the overall system performance. The feed-forward value alone can often provide the major portion of the controller output. The PID controller can then be used primarily to respond to whatever difference or *error* remains between the set point (SP) and the actual value of the process variable (PV). Since the feed-forward output is not affected by the process feedback, it can never cause the control system to oscillate, thus improving the system response and stability.

For example, in most motion control systems, in order to accelerate a mechanical load under control, more force or torque is required from the prime mover, motor, or actuator. If a velocity loop PID controller is being used to control the speed of the load and command the force or torque being applied by the prime mover, then it is beneficial to take the instantaneous acceleration desired for the load, scale that value appropriately and

add it to the output of the PID velocity loop controller. This means that whenever the load is being accelerated or decelerated, a proportional amount of force is commanded from the prime mover regardless of the feedback value. The PID loop in this situation uses the feedback information to effect any increase or decrease of the combined output in order to reduce the remaining difference between the process set point and the feedback value. Working together, the combined open-loop feed-forward controller and closed-loop PID controller can provide a more responsive, stable and reliable control system.

Another problem faced with PID controllers is that they are linear. Thus, performance of PID controllers in non-linear systems (such as HVAC systems) is variable. Often PID controllers are enhanced through methods such as PID gain scheduling or fuzzy logic. Further practical application issues can arise from instrumentation connected to the controller. A high enough sampling rate, measurement precision, and measurement accuracy are required to achieve adequate control performance.

A problem with the Derivative term is that small amounts of measurement or process noise can cause large amounts of change in the output. It is often helpful to filter the measurements with a low-pass filter in order to remove higher-frequency noise components. However, low-pass filtering and derivative control can cancel each other out, so reducing noise by instrumentation means is a much better choice. Alternatively, the differential band can be turned off in many systems with little loss of control. This is equivalent to using the PID controller as a *PI* controller.

]

## **2.9 PHYSICAL IMPLEMENTATION OF PID CONTROL**

In the early history of automatic process control the PID controller was implemented as a mechanical device. These mechanical controllers used a lever, spring and a mass and were often energized by compressed air. These pneumatic controllers were once the industry standard.

Electronic analog controllers can be made from a solid-state or tube amplifier, a capacitor and a resistance. Electronic analog PID control loops were often found within more complex electronic systems, for example, the head positioning of a disk drive, the power conditioning of a power supply, or even the movement-detection circuit of a modern seismometer. Nowadays, electronic controllers have largely been replaced by digital controllers implemented with microcontrollers or FPGAs.

Most modern PID controllers in industry are implemented in programmable logic controllers (PLCs) or as a panel-mounted digital controller. Software implementations have the advantages that they are relatively cheap and are flexible with respect to the implementation of the PID algorithm.

### 2.9.1 Ideal versus standard PID form

The form of the PID controller most often encountered in industry, and the one most relevant to tuning algorithms is the *standard form*. In this form the  $K_p$  gain is applied to the  $I_{out}$ , and  $D_{out}$  terms, yielding:

$$MV(t) = K_p \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de}{dt}(t) \right) \quad (2.6)$$

where

$T_i$  is the *integral time*

$T_d$  is the *derivative time*

In the ideal parallel form, shown in the controller theory section

$$MV(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt}(t) \quad (2.7)$$

the gain parameters are related to the parameters of the standard form through

$K_i = \frac{K_p}{T_i}$  and  $K_d = K_p T_d$ . This parallel form, where the parameters are treated as



simple gains, is the most general and flexible form. However, it is also the form where the parameters have the least physical interpretation and is generally reserved for theoretical treatment of the PID controller. The standard form, despite being slightly more complex mathematically, is more common in industry.

### 2.9.2 Laplace form of the PID controller

Sometimes it is useful to write the PID regulator in Laplace transform form:

$$G(s) = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s} \quad \text{.2.8}$$

Having the PID controller written in Laplace form and having the transfer function of the controlled system, makes it easy to determine the closed-loop transfer function of the system.

## 2.10 P I CONTROLLER

In control engineering, a **PI Controller** (proportional-integral controller) is a feedback controller which drives the plant to be controlled with a weighted sum of the error (difference between the output and desired set-point) and the integral of that value. It is a special case of the common PID controller in which the derivative (D) of the error is not used.

The controller output is given by

$$K_P \Delta + K_I \int \Delta dt \quad \text{.2.9}$$

where  $\Delta$  is the set-point error.

### 2.10.1 Advantages of a Proportional Plus Integral Controller

The integral term in a PI controller causes the steady-state error to be zero for a step input.

## 2.11 PI CONTROLLER MODEL

A PI controller can be modelled easily in software such as Simulink using a "flow chart" box involving Laplace operators:

$$C = \frac{G(1 + s\tau)}{sT} \quad \text{í 2.1.0}$$

where

$G = K_P =$  proportional gain

$G / T = K_I =$  integral gain

### 2.11.1 Finding a value for G

Setting a value for  $G$  is often a trade off between decreasing overshoot and increasing settling time.

### 2.11.2 Finding a value for $\tau$

Finding a proper value for  $\tau$  is an iterative process.

- 1) Set a value for  $G$  from the optimal range.
- 2) View the Nichols Plot for the open-loop response of the system. Observe where the response curve crosses the 0dB line. This frequency is known as the cross-over frequency ( $f_c$ ).
- 3) The value of  $\tau$  can be calculated as:

$$\tau = 1 / f_c$$

- 4) Decreasing  $\tau$  decreases the phase margin, however it eliminates a greater proportion of the steady-state errors.

## **2.12 DISADVANTAGES OF A PROPORTIONAL PLUS INTEGRAL CONTROLLER**

The problem with using a PI controller is that it introduces a phase-lag. This means that on a Nichols Plot, the stability margin (the phase margin) decreases. So careful design considerations with respect to the gain must be considered.

## **2.13 TYPES OF MOTOR CONTROLLERS**

Motor controllers can be manually, remotely or automatically operated. They may include only the means for starting and stopping the motor or they may include other functions.

An electric motor controller can be classified by the type of motor it is to drive such as permanent magnet, servo, series, separately excited, and alternating current.

A motor controller is connected to a power source such as a battery pack or power supply, and control circuitry in the form of analog or digital input signals.

### **2.13.1 Motor starters**

Main article: Direct on line starter

Main article: Motor soft starter

A small motor can be started by simply plugging it into an electrical receptacle or by using a switch or circuit breaker. A larger motor requires a specialized switching unit called a motor starter or motor contactor. When energized, a direct on line (DOL) starter immediately connects the motor terminals directly to the power supply. A motor soft starter connects the motor to the power supply through a voltage reduction device and increases the applied voltage gradually or in steps.

### 2.13.2 Adjustable-speed drives

Main article: Adjustable-speed drive.

An adjustable-speed drive (ASD) or variable-speed drive (VSD) is an interconnected combination of equipment that provides a means of driving and adjusting the operating speed of a mechanical load. An electrical adjustable-speed drive consists of an electric motor and a speed controller or power converter plus auxiliary devices and equipment. In common usage, the term "drive" is often applied to just the controller. [\[4\]\[5\]](#)

### 2.14 CALCULATING STEADY-STATE ERRORS

. Steady-state error can be calculated from the open or closed-loop transfer function for unity feedback systems. For example, let's say that we have the following system:

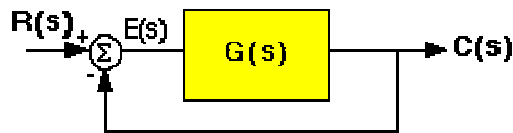


Fig 2.5

which is equivalent to the following system:

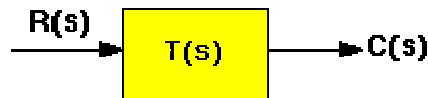


Fig2.6

We can calculate the steady state error for this system from either the open or closed-loop transfer function using the final value theorem (remember that this theorem can only be applied if the denominator has no poles in the right-half plane):

$$e(\infty) = \lim_{s \rightarrow 0} \frac{sR(s)}{1 + G(s)} \quad \dots 2.1.1$$

$$e(\infty) = \lim_{s \rightarrow 0} sR(s) [1 - T(s)] \quad \dots 2.1.2$$

Now, let's plug in the Laplace transforms for different inputs and find equations to calculate steady-state errors from open-loop transfer functions given different inputs:

- Step Input ( $R(s) = 1/s$ ):

$$e(\infty) = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)} = \frac{1}{1 + K_p} \Rightarrow K_p = \lim_{s \rightarrow 0} G(s) \quad \text{í í í í í í 2.1.3}$$

- Ramp Input ( $R(s) = 1/s^2$ ):

$$e(\infty) = \frac{1}{\lim_{s \rightarrow 0} sG(s)} = \frac{1}{K_v} \Rightarrow K_v = \lim_{s \rightarrow 0} sG(s) \quad \text{í í í í í í 2.1.4}$$

- Parabolic Input ( $R(s) = 1/s^3$ ):

$$e(\infty) = \frac{1}{\lim_{s \rightarrow 0} s^2 G(s)} = \frac{1}{K_a} \Rightarrow K_a = \lim_{s \rightarrow 0} s^2 G(s) \quad \text{í í í 2.1.5}$$

When we design a controller, we usually want to compensate for disturbances to a system. Let's say that we have the following system with a disturbance:

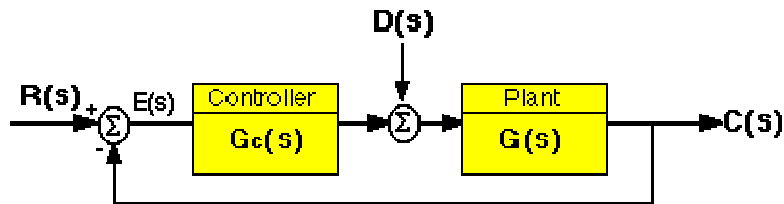


Fig 2.7

we can find the steady-state error for a step disturbance input with the following equation:

$$e(\infty) = \frac{1}{\lim_{s \rightarrow 0} \frac{1}{G(s)} + \lim_{s \rightarrow 0} G_r(s)} \quad \text{í í í í í í í í í í í í í í í 2.1.6}$$

Lastly, we can calculate steady-state error for non-unity feedback systems:

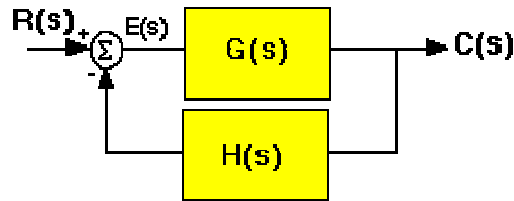


Fig 2.8

By manipulating the blocks, we can model the system as follows:

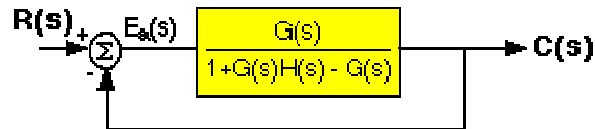


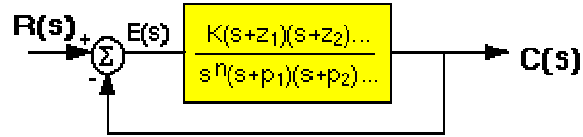
Fig 2.9

Now, simply apply the equations we talked about above.

## 2.15 SYSTEM TYPE AND STEADY-STATE ERROR

If you refer back to the equations for calculating steady-state errors for unity feedback systems, you will find that we have defined certain constants (known as the static error constants). These constants are the position constant ( $K_p$ ), the velocity constant ( $K_v$ ), and the acceleration constant ( $K_a$ ). Knowing the value of these constants as well as the system type, we can predict if our system is going to have a finite steady-state error.

First, let's talk about system type. The system type is defined as the number of pure integrators in a system. That is, the system type is equal to the value of  $n$  when the system is represented as in the following figure:



**Fig 2.1.0**

Therefore, a system can be type 0, type 1, etc. Now, let's see how steady state error relates to system types:

<u><b>Type 0 systems</b></u>	<i>Step Input</i>	<i>Ramp Input</i>	<i>Parabolic Input</i>
<i>Steady State Error Formula</i>	$1/(1+K_p)$	$1/K_v$	$1/K_a$
<i>Static Error Constant</i>	$K_p = \text{constant}$	$K_v = 0$	$K_a = 0$
<i>Error</i>	$1/(1+K_p)$	infinity	infinity

Table 2.3

<b>Type 1 systems</b>	<i>Step Input</i>	<i>Ramp Input</i>	<i>Parabolic Input</i>
<i>Steady State Error Formula</i>	$1/(1+K_p)$	$1/K_v$	$1/K_a$
<i>Static Error Constant</i>	$K_p = \text{infinity}$	$K_v = \text{constant}$	$K_a = 0$
<i>Error</i>	0	$1/K_v$	infinity

**Table 2.**

<b>Type 2 systems</b>	<i>Step Input</i>	<i>Ramp Input</i>	<i>Parabolic Input</i>
<i>Steady State Error Formula</i>	$1/(1+K_p)$	$1/K_v$	$1/K_a$
<i>Static Error Constant</i>	$K_p = \text{infinity}$	$K_v = \text{infinity}$	$K_a = \text{constant}$
<i>Error</i>	0	0	$1/K_a$

**Fig 2.5**



## CHAPTER THREE

### 3.0 OBJECTIVES

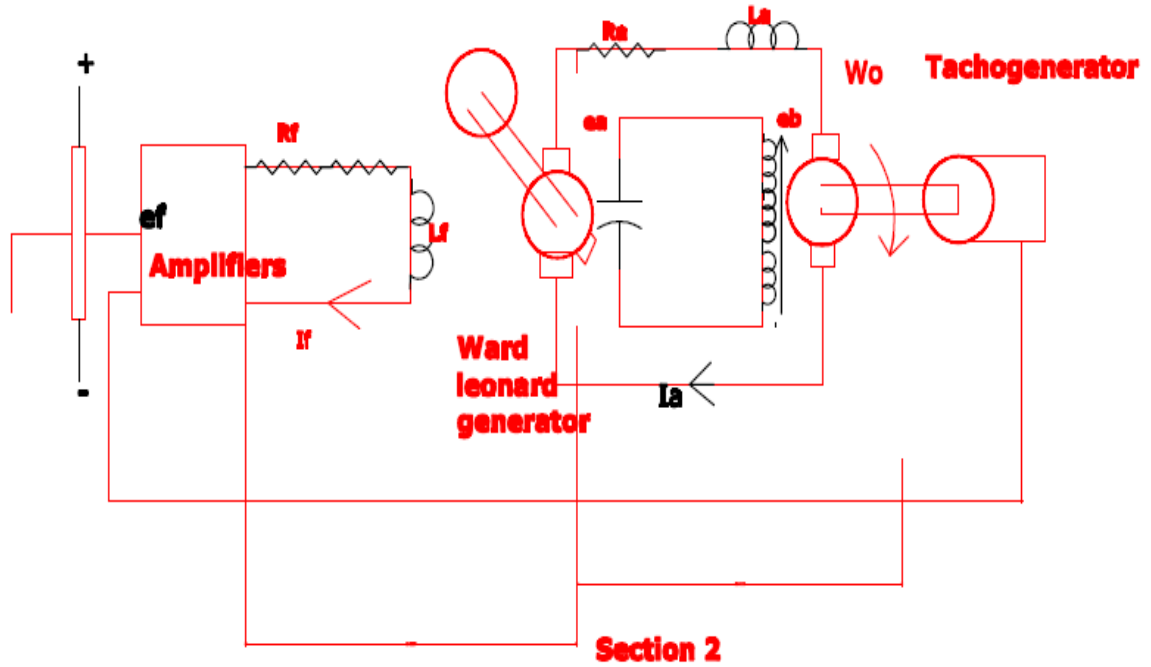
- ✚ To use a proportional action that will reduce steady state error and increase the step response overshoot as the proportional band  $K_p$  is reduced.
- ✚ Adjustment of compensator parameters equivalent to the tuning of general purpose process controllers.
- ✚ Use of regenerative braking which requires that the directional sense of the motor torque be controlled in a manner causing a return of energy to the AC supply when this is required
- ✚ A negative armature current feedback path in cooperating a dead space element may be provided to affect automatic armature current limiting so as to protect the motor against possible overload.
- ✚ Integral action that will eliminate steady state error arising from most causes and as the integral action time  $T_I$  is reduced ,increase the step response overshoot .
- ✚ Negative armature current feed back that can be used to change the converter into to a current source as opposed to voltage source.
- ✚ A derivative action that will reduce the step response overshoot as the derivative action  $T_d$  is increased.
- ✚ The primary (negative) speed feed back to be obligatory and normally derived from a DC tachogenerator.

### 3.1 ANALYSIS OF THE GENERAL CLOSED LOOP WARD LEONARD SPEED CONTROL SYSTEM

The schematic diagram of a closed Ward Leonard Speed Control System is shown in Fig. 2. Although the title of this project is Ward Leonard speed control system and an open loop Ward Leonard System do exist, the project is strictly concerned with the closed loop Ward Leonard system as has already been quietly implied in the introduction.

As shown in Fig. 2 the system can be divided into two sections for purposes of analysts.

### Driving 3-phase motor



### Section 1

**FIG:1 THE GENERAL CLOSED-LOOP WARD LEONARD SPEED CONTROL SYSTEM**

Itemref	Quantity	Title/Name, designation, material, dimension etc			Article No./Reference	
Designed by SEBASTIAN MUTHUSI	Checked by JEREMIAH	Approved by - date 20/05/2009	File name SCHEMATIC DIAGRAM OF WARD LEONARD SPEED CONTROL SYSTEM	Date	Scale	
DR MANG'OLI			ELECTRICAL DIAGRAM			
			N01		Edition 2009	Sheet 1/1

### 3.2 CONSIDER SECTION 1

The following equations can be written down

$$L_f \frac{di_f}{dt} + R_f i_f = e_f \tag{1}$$

$$e_a = k_f i_f$$

Where

$L_f$  = Field winding inductance

$i_f$  = field current

$R_f$  = field winding resistance

$e_f$  = voltage across the field winding.

$e_A$  = voltage output of the W.L. generator.

Combining equations 1 and 2, and through Laplace transformation, we have.

$$\frac{e_a(S)}{e_f(S)} = \frac{k_g}{(1 + T_g S)} \tag{3}$$

Where  $K_g = \frac{k_f}{R_f}$

= W.L. Generator gain constant.

$$T_g = \frac{L_f}{R_f} = \text{W.L. generator excitation time constant.}$$

### 3.3 NOW CONSIDER SECTION II

The following equation can be written down.

$$e_b = e_a - \left( R_a i_a + \frac{L_a di_a}{dt} \right) \tag{4}$$

$$e_b = k_b \omega(t) \tag{5}$$

$$T = K_T i_a = F \omega(t) + J d \frac{\omega(t)}{dt} \tag{6}$$

Where in the above equations,

$e_b$  = work Motor Back emf

$e_a$  = W.L. Generator output voltage

$R_a$  = Armature resistance of Work motor

$L_a$  = Armature inductance of Work motor

$i_a$  = Armature current

$K_b$  = Back constant

$K_T$  = Torque constant

$F$  = Friction

$J$  = Inertia of Work motor

$W_0(t)$  = output speed

By combining equations (4), (5), (6) and taking Laplace transforms, we arrive at

$$\frac{W_0(s)}{e_a(s)} = \left[ \frac{1}{s^2 \frac{L_a J}{K_T} + s \frac{(R_a J + F L_a)}{K_T} + \frac{F R_a + K_b}{K_T}} \right] \dots \dots \dots .7$$

### 3.4 ASSUMPTION

For all practical purpose we can neglect  $L_a$ . Firstly because it is small in comparison with  $K_T$  even when the motor is on load. Secondly because the frequency of operation in dc control systems is of the order of a few radians per sec

Hence

$$\frac{W_0(s)}{e_a(s)} = \frac{K_T}{(s R_a J + F R_a + K_b K_T)} = \frac{K_m}{(1 + T_m s)} \dots \dots \dots .8$$

$$K_m = \frac{K_T}{F R_a + K_b K_T} = \text{Work motor gain constant}$$

$$T_m = \frac{R_a J}{F R_a + K_b K_T} = \text{Work motor mechanical time constant}$$

### 3.5 CONSIDER THE FEED-BACK LOOP

The following is true:

$$e_0 = K_t W_0(t) \dots \dots \dots .9$$

$e_0$  = output voltage from tachogenerator

$K_t$  = tachogenerator constant

$$e = (e_{in} - e_0) \dots\dots\dots 10$$

$e$  = error voltage

$e_{in}$  = reference voltage

$e_0$  = output voltage from tachogenerator

### 3.6 THE AMPLIFIERS

Assuming that the time constants of the amplifiers are small in comparison with the time constants of the motor and generator, we can write

$$e_f = K_A e_{in} \dots\dots\dots 11$$

Where  $K_A$  = gain of the amplifiers.

### 3.7 THE TRANSFER FUNCTION BLOCK DIAGRAM

We are now in a position to make a transfer function block diagram. By combining equations 3, 8, 9 and 11 we obtain the transfer function Block diagram shown in fig 2.

### 3.8 THE CLOSED LOOP TRANSFER FUNCTION AND THE CHARACTERISTICS EQUATION

From Fig. 3 the open loop transfer-function

$$G(s) = \frac{K_A K_g K_m}{(1 + T_g S)(1 + T_m S)} \dots\dots\dots 12$$

It can be shown that the closed loop transfer function is given by

$$\frac{W_0(s)}{e_{in}(s)} = \frac{G(s)}{1 + G(s)H(S)} \dots\dots\dots 13$$

Where  $H(S)$  is the feedback loop transfer function.

In our case

$$H(s) = k_t$$

Hence

$$\frac{W_0(s)}{e_a(s)} = \frac{\frac{K_A K_g K_m}{(1+T_g S)(1+T_m S)}}{\frac{1+K_A K_g K_m K_t}{(1+T_g S)(1+T_m S)}}$$

$$= \frac{K_A K_g K_m}{T_m T_g S^2 + (T_m + T_g)S + 1 + K_A K_m K_g K_t} \quad \dots 14$$

Thus the characteristic equation is given by:-

$$S^2 + \frac{(T_m + T_g)S}{T_m T_g} + \frac{1 + K_A K_m K_g K_t}{T_m T_g} = 0 \quad \dots 15$$

### 3.9 EXAMINATION FOR STABILITY BY THE ROUTH HURWITZ CRITERION

From equation 15, we can assemble the Routh Hurwitz Array:

$S^2$	1	$\frac{1 + K_A K_m K_g K_t}{T_m T_g}$
$S^1$	$\frac{T_m + T_g}{T_m T_g}$	0
$S^0$	$\frac{(1 + K_A K_m K_g K_t)(T_m + T_g)}{(T_m T_g)^2}$	

For stability

$$\frac{(1 + K_A K_m K_g K_t)(T_m + T_g)}{(T_m T_g)^2} > 0 \quad \dots 16$$

$$1 + K_A K_m K_g K_t > 0$$

$$K_A K_m K_g K_t > -1$$

$$K_A > \frac{-1}{K_m K_g K_t} \quad \dots 17$$

This means that absolute stability is guaranteed for all values of the amplifier gain  $K_A$ .

### 3.10 THE SYSTEM LOOP GAIN

By definition Loop gain is

$$K = \lim_{S \rightarrow 0} G(s)H(s) \quad \dots 18$$

Where

G(s) = Open loop transfer function

H(s) = Feed back loop transfer function

In case of the Ward Leonard speed control system this becomes

$$K = K_A K_g K_m K_t \quad \dots 19$$

### 3.11 THE POSITIONAL CONSTANT $K_p$

By definition

$$\begin{aligned} K_p &= \lim_{S \rightarrow 0} G(s)H(s) \\ &= K_A K_g K_m K_t \quad \dots 20 \\ &= K \text{ (for the system under consideration)} \end{aligned}$$

#### 3.11.1 Velocity Constant $K_v$

By definition

$$\begin{aligned} K_v &= \lim_{S \rightarrow 0} S G(s)H(s) \quad \dots 21 \\ &= \lim_{S \rightarrow 0} \frac{S(K_A K_g K_m K_t)}{(1 + T_g S)(1 + T_m S)} \\ &= 0 \end{aligned}$$

### 3.12 ACCELERATION CONSTANT OF THE SYSTEM

$$\begin{aligned} K_{ac} &= \lim_{S \rightarrow 0} S^2 G(s)H(s) \\ &= \lim_{S \rightarrow 0} S^2 \frac{K_A K_g K_m K_t}{(1 + T_g S)(1 + T_m S)} \quad \dots 22 \end{aligned}$$

$$= 0$$

### 3.13 STEADY STATE ERROR

$$Ss(t) = \lim_{s \rightarrow 0} \frac{SR(s)}{1 + G(s)H(s)} \quad \dots 23$$

Where R(s) is the input constant

Thus for a unit step input,

$$\begin{aligned} ss(t) &= \lim_{s \rightarrow 0} \frac{S}{S(1 + G(s)H(s))} \\ &= \frac{1}{1 + K_A K_g K_m K_t} \quad \dots 24 \\ &= \frac{1}{1 + K_p} \end{aligned}$$

While for a unit ramp

$$\begin{aligned} ess(t) &= \lim_{s \rightarrow 0} \frac{S}{S^2(1 + G(s)H(s))} \quad \dots 25 \\ &= \infty \end{aligned}$$



### 3.14 EXAMINATION FOR THE NATURAL FREQUENCY $\omega_n$ OF THE SYSTEM

From Equation

$$\frac{W_o(s)}{e_{in}(s)} = \frac{\frac{K_A K_g K_m}{T_m T_g}}{S^2 + \frac{(T_m + T_g)s}{T_m T_g} + \frac{(1 + K_A K_g K_m K_t)}{T_m T_g}}$$

$$= \frac{K_0}{S^2 + 2\varepsilon\omega_n S + \omega_n^2} \quad \text{í 26}$$

where  $\varepsilon$  ó damping factor

$\omega_n$  ó natural frequency of the system

$$K_0 = \frac{(K_A K_m K_g)}{T_m T_g}$$

From the above we have

$$\omega_n^2 = \frac{1 + K_A K_g K_m K_t}{T_m T_g}$$

$$\omega_n = \left[ \frac{(1 + K_A K_g K_m K_t)}{T_m T_g} \right]^{\frac{1}{2}} \quad \text{í 27}$$

### 3.15 DETERMINATION OF THE TIME DOMAIN RESPONSE OF THE SYSTEM AND THE RISE TIME EQUATION

From equation 26

$$W_o(s) = \frac{K_0}{(S + \alpha)(s + \gamma)} \quad \text{í ..28}$$

Where

$$K_0 = \frac{K_A K_g K_m}{T_m T_g}$$

$$-s_1 = w_n + jw_n \sqrt{1 - \epsilon^2} = \delta + jw \dots ..29$$

$$-s_2 = w_n - jw_n \sqrt{1 - \epsilon^2} = \delta - jw \dots ..30$$

Hence

$$W = W_n \sqrt{1 - \epsilon^2} \dots .. 31$$

Let us assume a unit step input subjected into the system

$$\frac{W_0(s)}{e_{in}(s)} = \frac{K_0}{(S + \alpha)(s + \gamma)}$$

$$e_{in}(s) = \frac{1}{s}$$

Hence

$$W_0(s) = \frac{K_0}{S(s + \alpha)(s + \gamma)} \dots ..32$$

$$W_0(s) = \frac{K_0}{\alpha\gamma} + \frac{K_0}{\gamma(\gamma - \alpha)(s + \gamma)} + \frac{\alpha}{\alpha(\alpha - \gamma)(s + \alpha)}$$

$$W_0(t) = \frac{K_0}{\alpha\gamma} \left[ 1 + \frac{\gamma}{(\alpha - \gamma)} e^{-(\delta + jw)t} + \frac{\alpha}{(\gamma - \alpha)} e^{-(\delta - jw)t} \right]$$

$$W_0(t) = \frac{K_0}{\alpha} 1 - e^{-\delta t} \left( \cos wt + \frac{\delta}{w} \sin wt \right) \dots .. 33$$

To determine where  $W_0(t)$  is a maximum we differentiate equation 33 with respect to time and equate the derivative to zero.

$$\frac{K_0}{\alpha\gamma} \left[ e^{-\delta t} \left( \delta \cos wt + \frac{\delta^2}{w} \sin wt \right) - e^{-\delta t} (-w \sin wt + \delta \cos wt) \right] = 0$$

$$\left[ \frac{\delta^2}{w^2} + 1 \right] \sin wt = 0$$

Since in general  $\left[ \frac{\delta^2}{\omega^2} + 1 \right] \neq 0$

$$\sin \omega t_m = 0$$

$$\text{i.e. } \omega t_m = \pi$$

Using the conventional definition of rise time in control  $t_m$  i.e. the time it takes a system to get to the 1st overshoot we have

$$\omega t_r = \pi$$

$$t_r = \frac{\pi}{\omega} = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} \quad .34$$

### 3.16 EFFECTS OF LOAD TORQUE ON THE SYSTEM

The effect of load torque on the Work-motor is to reduce the speed of the said motor. This reduction should be transmitted to the Ward Leonard generator and consequently to the driving prime mover. Unless the prime-mover is such a machine as a synchronous motor the net effect will be a reduction in the speed of the prime-mover. In the Ward-Leonard system this is a high undesirable state. In fact the foregoing analysis has assumed a constant speed implicitly. A variable speed in the prime mover would mean a varying Kg. This is obviously a non linearity in the system. Nevertheless, induction motors are used as prime movers in the system almost invariably. This is achieved as we shall show below by providing high gain in the amplifiers and the use of Proportional Integral Controllers.

#### 3.16.1 Consider Fig. 3

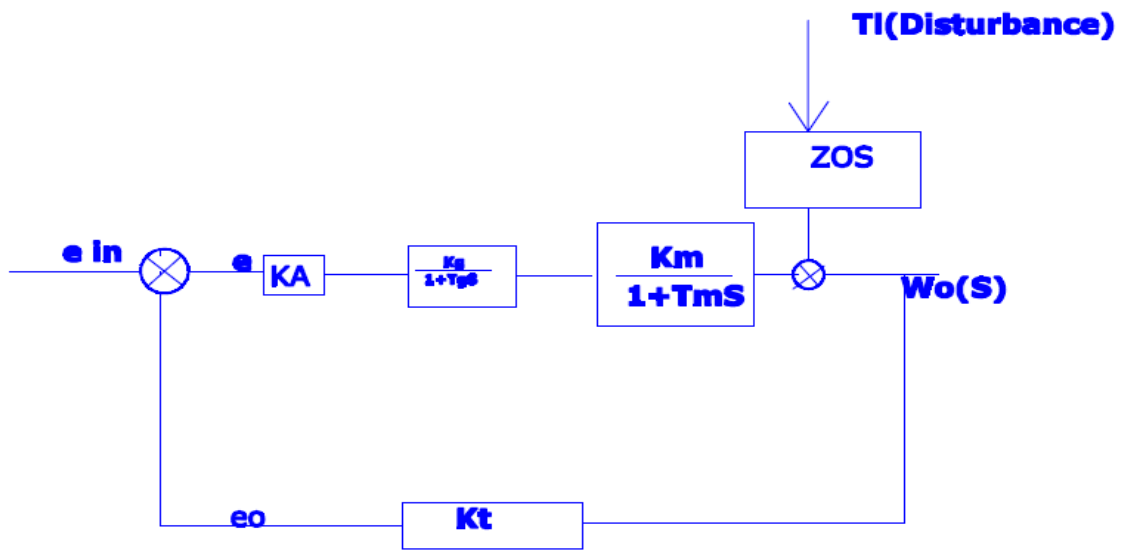
The load torque could be considered as a disturbance into the system. Considering now that the dotted line is full and  $Z_0(s)$  the disturbance transfer function forms part of the system we see that:

The open loop transfer function as seen at the point from which the disturbance enters into the system is given by

$$\left[ \frac{W_0(s)}{T_L} \right]_{OL} = Z_0(s) \quad \text{35}$$

The closed loop transfer function as seen from the same point is given by

$$\left[ \frac{W_0(s)}{T_L} \right]_{FB} = \frac{Z_0(s)}{1 + \frac{K_A K_m K_g K_t Z_0(s)}{(1 + T_m s)(1 + T_g s)}} \quad \text{36}$$



**KA=Amplifiers gain**  
**e=error voltage**  
**Tg=generator excitation time constant**  
**Tm=motor mechanical time constant**  
**Kt=tachogenerator time constant**  
**em=Reference voltage**  
**Kg=Generator gain constant**  
**Km=motor gain constant**

**FIG3:TRANSFERBLOCK DIAGRAM OF SIMPLIFIED CLOSED-LOOP WARD LEONARD SPEED CONTROL SYSTEM**

Itemref	Quantity	Title/Name, designation, material, dimension etc			Article No./Reference	
Designed by sebastian	Checked by jeremiah	Approved by - date 20/05/2009	File name transfer block diagram	Date 20/05/2009	Scale 1:1	
Dr Mango'li			electrical diagram			
			no 3		Edition 2009	Sheet 1/1

### 3.17 INDEX OF CONTROL

The index of control is defined as

$$\frac{\left[ \frac{W_0(s)}{T_L} \right]_{FB}}{\left[ \frac{W_0(s)}{T_L} \right]_{OL}} = \frac{1}{1 + \frac{K_A K_m K_g K_t Z_0(s)}{(1 + T_m S)(1 + T_g S)}} \quad \text{í í í í í í í í í í í í í í 37}$$

The index of control represents a measure of disturbance reduction by the use of feedback.

It is self evident that by increasing  $K_A$ , the gain of the amplifiers, we increase considerably the disturbance reduction. It is also evident that no matter how large  $K_A$  is, the effect of disturbances into will never be eliminated. To eliminate the residual effects of disturbances completely we use integration in the system. Hence the importance of Proportional Integral controllers.

### 3.18 MEASUREMENT AND DESIGN OF CONTROL SYSTEM

With the foregoing analysis of the closed loop ward Leonard speed control system fairly complete, we are now in a position to decide on what measurement to make on the project system to help us design a speed control unit.

The constants relevant to design are  $K_g, K_m, K_t, T_m, T_g$  and  $R_f$ . field winding resistance of the ward Leonard generator. By determining  $K_g, K_m, K_t$  and by choice of suitable  $K$ -loop gain we shall be in a position to choose a suitable  $K_A$  such that it is consonant with the demand for good disturbance reduction. The knowledge of  $T_g$  and  $T_m$  will help us decide of suitable location of a PI controller in the frequency domain whereas knowing  $R_f$  we are able to decide on the supply voltage necessary to achieve a required current in the ward Leonard generator fields.

#### 3.18.1 Determination of $K_g$

This constant was determined by taking some voltage measurement across the field winding of the generator and at the same time taking the measurement of the corresponding generated voltage from the W.L Generator.

These measurements of the field winding voltage were plotted against the generator output voltage. The slope gives  $K_g$ . see graph NO 1:

### **3.18.2 Determination of $K_m$**

For various voltages inputs to the work motor the corresponding speeds were taken. A plot of input voltage versus output speed gives  $K_m$ . See graph NO 2

### **3.18.3 Determination of $K_t$**

For various speed of the work motor the output voltage at the tachogenerator terminals was measured and the speed was plotted against the output voltage. The slope gives  $K_t$ . See graph NO.3;

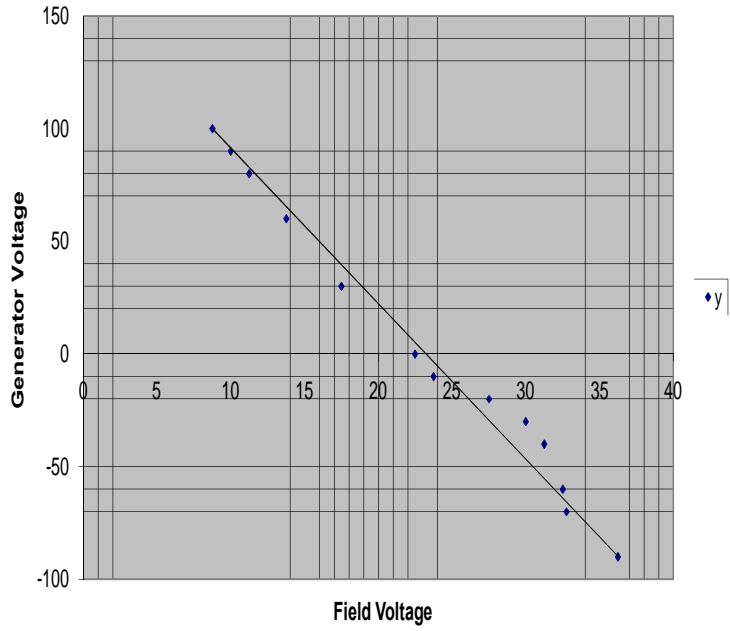
### **3.18.4 Determination of the field resistance**

The field resistance was determined firstly by the use of an avometer. This gave a value of about 150 ohms per half field. This was not realistic. Consequently determination of the same  $R_f$  by the dc drop test gave a half field resistance of about 30 ohms .the plot of the voltage versus current is shown in graph NO.4:

### **3.18.5 Determination of $T_g$ and $T_m$**

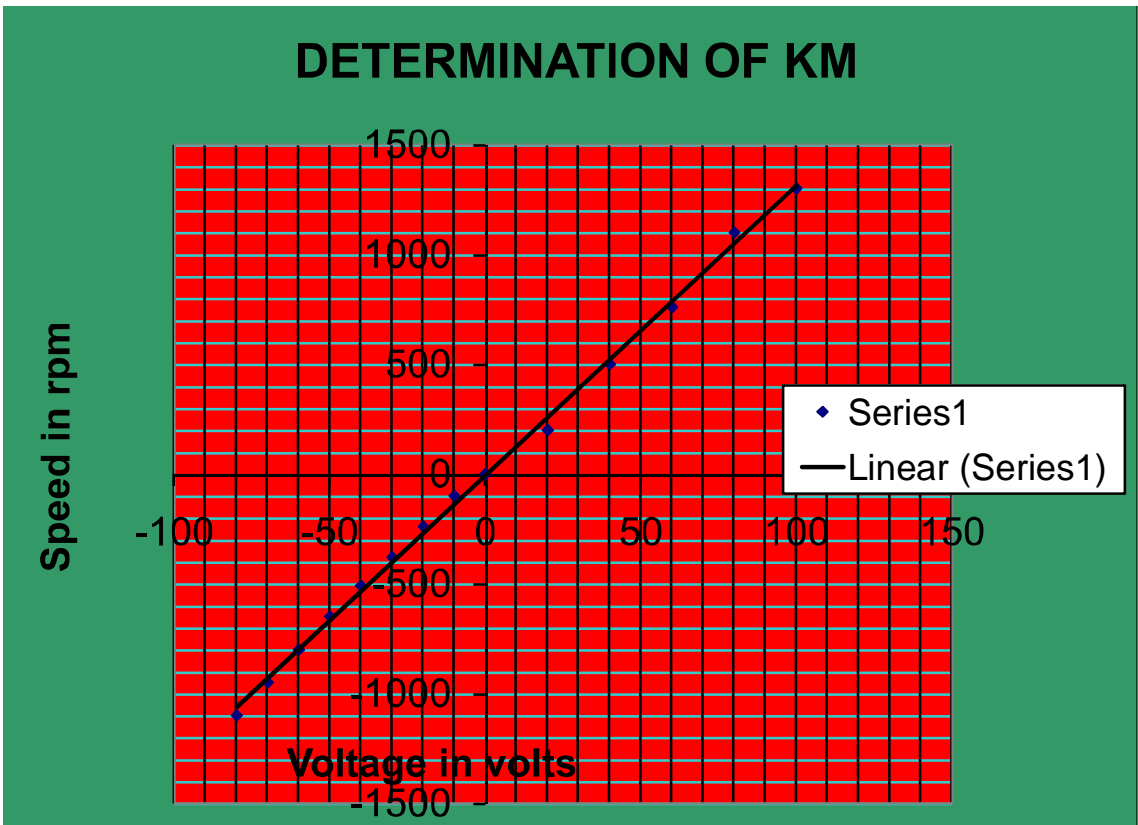
Theoretically this values where to be obtained by the use of a cathode ray oscilloscope, their trace photographed and a tangent drawn from the origin and where it intersects the horizontal line indicating the magnitude of the unit step. At the ward Leonard speed control in the Kenya airport authority this values were obtained direct from the machine.

### DETERMINATION OF KG



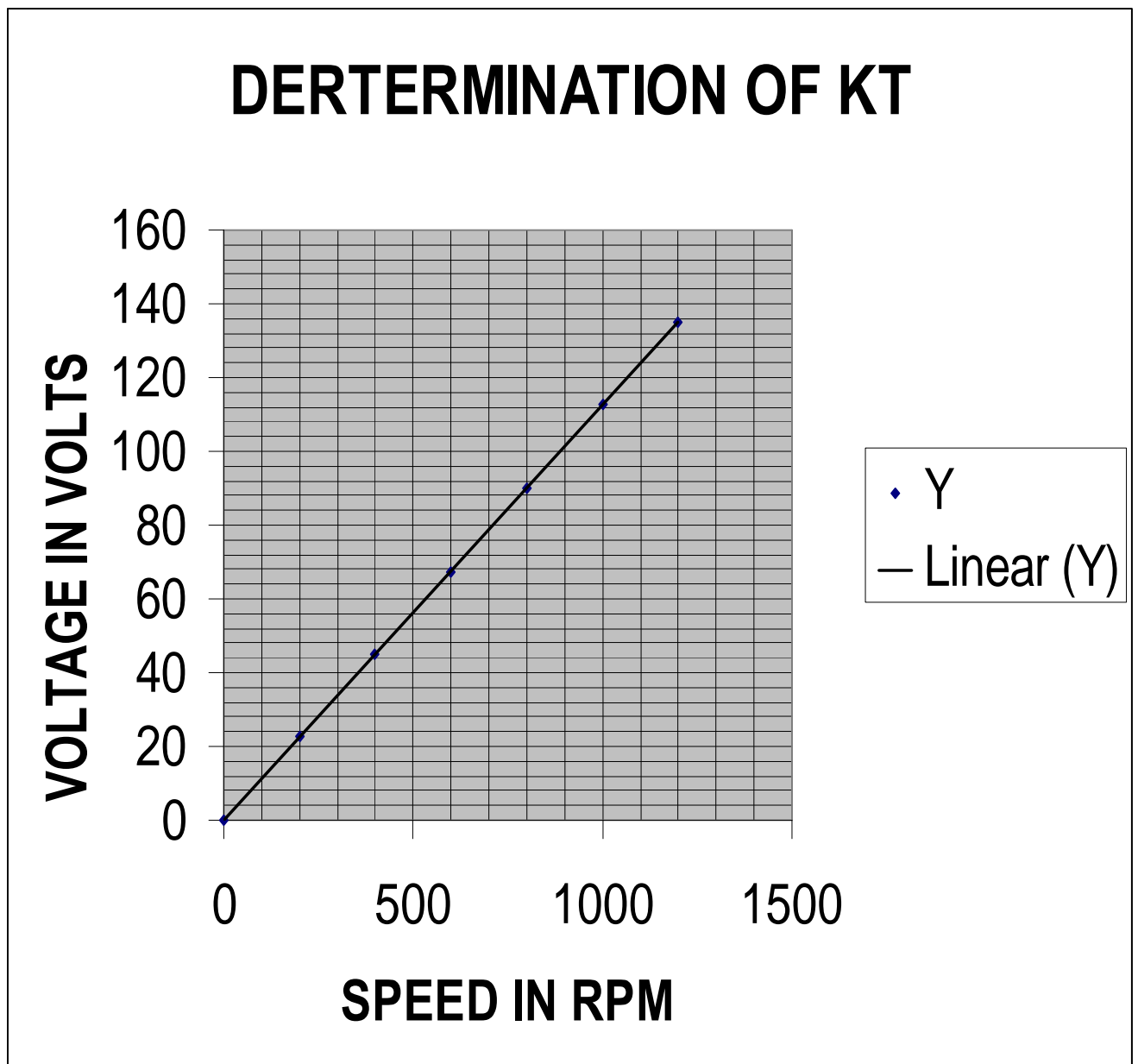
Graph no. 1



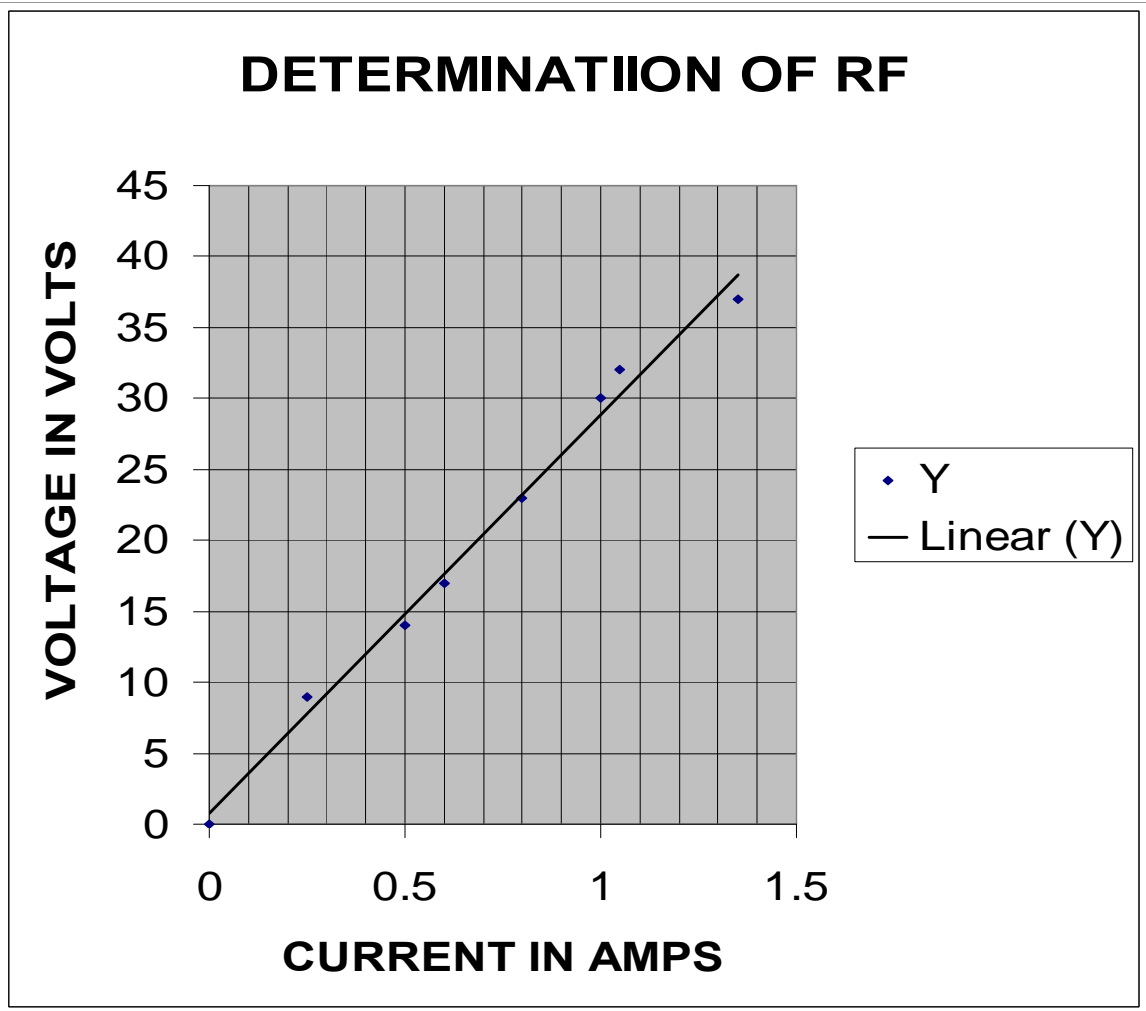


Graph no. 2

# DETERMINATION OF $K_T$



Graph no. 3



Graph no 4

## CHAPTER FOUR

### 4.0 SUMMARY OF RESULTS

$$K_g = 6.67$$

$$K_m = 1.35 \text{ radians/volt/sec.} \quad T_m = 70\text{msecs}$$

$$K_t = 1.05\text{volts/radian/sec} \quad T_g = 700\text{msecs}$$

$$R_f = 29.5 \text{ ohms}$$

### 4.1 DESIGN OF THE PROJECT SYSTEM

In this section we want to determine the minimum value of  $K_A$  ó the amplifier gain which makes the system have a fairly good response when the system is on non-load. Obviously we shall have to provide a much large  $K_A$  than this in order to absorb damping associated with loading in the design of amplifiers. We shall also confirm the result on stability reached earlier on in equation 17 by using the Routh Harwitz criterion. This time we shall use the Root locus for confirmation.

### 4.2 CHOICE OF THE DAMPING FACTOR

The damping factor of 0.425 was chosen because it gives an overshoot of approximately 22.88% and a good rise time of about 0.347 secs. In fact past experience has shown that this damping factor is within the optimum range of sampling factors for second order system.

### 4.3 CHOICE OF LOOP GAIN AND CONSEQUENTLY THE SETTLING ERROR

From equation 26 we have

$$\varepsilon = \frac{T_m T_g}{2T_m T_g W_n} = \frac{T_m + T_g}{2T_m T_g} x \frac{1}{\left[ \frac{1+k}{T_m T_g} \right]^{\frac{1}{2}}} \dots 37$$

$$\text{Putting } \zeta = 0.425$$

$$T_m = 70 \text{ msec}$$

$$T_g = 700 \text{ msec}$$

In equation 37 we obtain

$$K = 15.85 \cong 16$$

From Equation 24

$$\text{Setting error} = \frac{1}{1+K} = 0.595 < 6\%$$

#### **4.4 CHOICE OF $K_t$ AND HENCE THE VOLTAGE DIVIDING RATIO OF THE TACHOGENERATOR VOLTAGE.**

The value we obtained for  $K_t$  was 1.05 volts/radian/se/

At 1500 rpm the output from the tachogenerator is given by

$$\begin{aligned} \text{Output volts} &= \frac{1.05 \times 1500 \times 2\pi}{60} \\ &= 165\text{V} \end{aligned}$$

It is quite evident that it is unpalatable to work with such voltages closely. It would also mean that one would have to provide a reference voltage of  $\pm 165\text{V}$  to be able to operate at 1500rpm. It was therefore decided to reduce the output voltage from the tachogenerator to  $\frac{1}{16}$  of its value so that a reference voltage of approximately  $\pm 10\text{V}$  could be used.

Hence

$$K_t \phi = \frac{K_t}{16} 2 = \frac{1.05}{16} = 0.0657$$

Output from tachogenerator at 1500rpm

$$= \frac{165}{16} = 10.3 \text{ volts}$$

This reduction in the output voltage from tachogenerator is to be effected by means of a voltage divider.

## 4.5 CHOICE OF AMPLIFIER GAIN $K_A$

From equation 19

$$K = K_A K_g K_m K_t = 16$$

Since we cannot alter  $K_m$ ,  $K_g$  if we want the loop sign to remain at 16 after we have altered the tachogenerator constant we must alter  $K_A$ .

The altered  $K_A$  we designate as  $K_A'$ .

So we have

$$16 = K_A K_g K_m K_t = K_A' K_g K_m K_t'$$

$$K_A' = \frac{16}{1.35 \times 6.67 \times 0.65} = 27$$

## 4.6 THE TRANSFER FUNCTION OF THE PROJECT SYSTEM

From equation 14

$$\begin{aligned} \frac{W_o(s)}{in(s)} &= \frac{K_A K_g K_m}{T_m T_g S^2 + (T_m + T_g)S + 1 + K_A K_g K_m K_t} \\ &= \frac{K_A K_g K_m}{T_m T_g S^2 + (T_m + T_g)S + 1 + K_A' K_g K_m K_t'} \end{aligned} \quad .38$$

Feeding in the values of  $K_A'$ ,  $K_m$ ,  $K_g$ ,  $K_t'$ ,  $K_g$  and  $T_m$  in the above equation and rearranging we arrive at

$$\frac{W_o(s)}{e_{in}(s)} = \frac{975}{S^2 + 15.7S + 347} \quad .39$$

$$= \frac{975}{(S + 7.85 + j16.875)(s + 7.85 - j16.875)} \quad .40$$

## 4.7 CONFIRMATION OF ABSOLUTE STABILITY BY ROOT LOCUS

From equation 38 the characteristics equation can be written as

$$S^2 + 15.7S + 20.4(1+K) = 0$$

$$S = \frac{-15.7}{2} \pm \sqrt{\frac{298 - 81.6(1+K)}{4}} \quad \text{.41}$$

Quite clearly for  $K < -1$  we have a pole in the right half plane confirming instability as predicted by Routh Harwitz in equation 17. The complete Root Locus is shown in Graph No. 5A.

## 4.8 TIME DOMAIN SOLUTION OF SYSTEM RESPONSE TO A UNIT STEP FUNCTION

From Equation 33

$$W_o(f) = \frac{K_0}{\alpha\gamma} \left[ 1 - e^{-\delta t} \left( \cos wt + \frac{\delta}{w} \sin wt \right) \right]$$

Rearranging this equation, we arrive at

$$W_o(t) = \frac{K_0}{\alpha\gamma} \left[ 1 - e^{-\delta t} \frac{\delta^2 + w^2}{w} \sin(wt + \phi) \right] \quad \text{.42}$$

$$\text{Where } \phi = \tan^{-1} \frac{w}{\delta} \quad \text{.43}$$

$$\text{a) } K_0 = \frac{K_A K_g K_m}{T_g T_m} = 975 \quad \text{.44}$$

$$\text{b) } = \frac{(1 + K_A' K_g K_m K_t')}{T_m T_g} \quad \text{.45}$$

$$\text{c) } W_n = \frac{(1 + K_A K_g K_m K_t')^{\frac{1}{2}}}{T_m T_g} = 18.65 \text{ rad/sec} \quad \text{.46}$$

$$\text{d) } w = w_n \sqrt{1 - \varepsilon^2}$$

$$= 18.65 \times 0.82 = 16.8 \text{ rad/sec} \quad \text{.47}$$

$$e) \delta = \varepsilon w_n$$

$$= 0.425 \times 18.65 = 7.91 \text{ s} \quad \text{48}$$

Putting the above values in equation 42 and simplifying, we obtain

$$W_o(t) = 2.8 \left[ 1 - 1.11 e^{-7.85t} \sin 2\pi(16.8t + 0.18) \right] \text{ s} \quad \text{.49}$$

A plot of  $\frac{W_o(t)}{2.8}$  is shown in Graph No.5 C

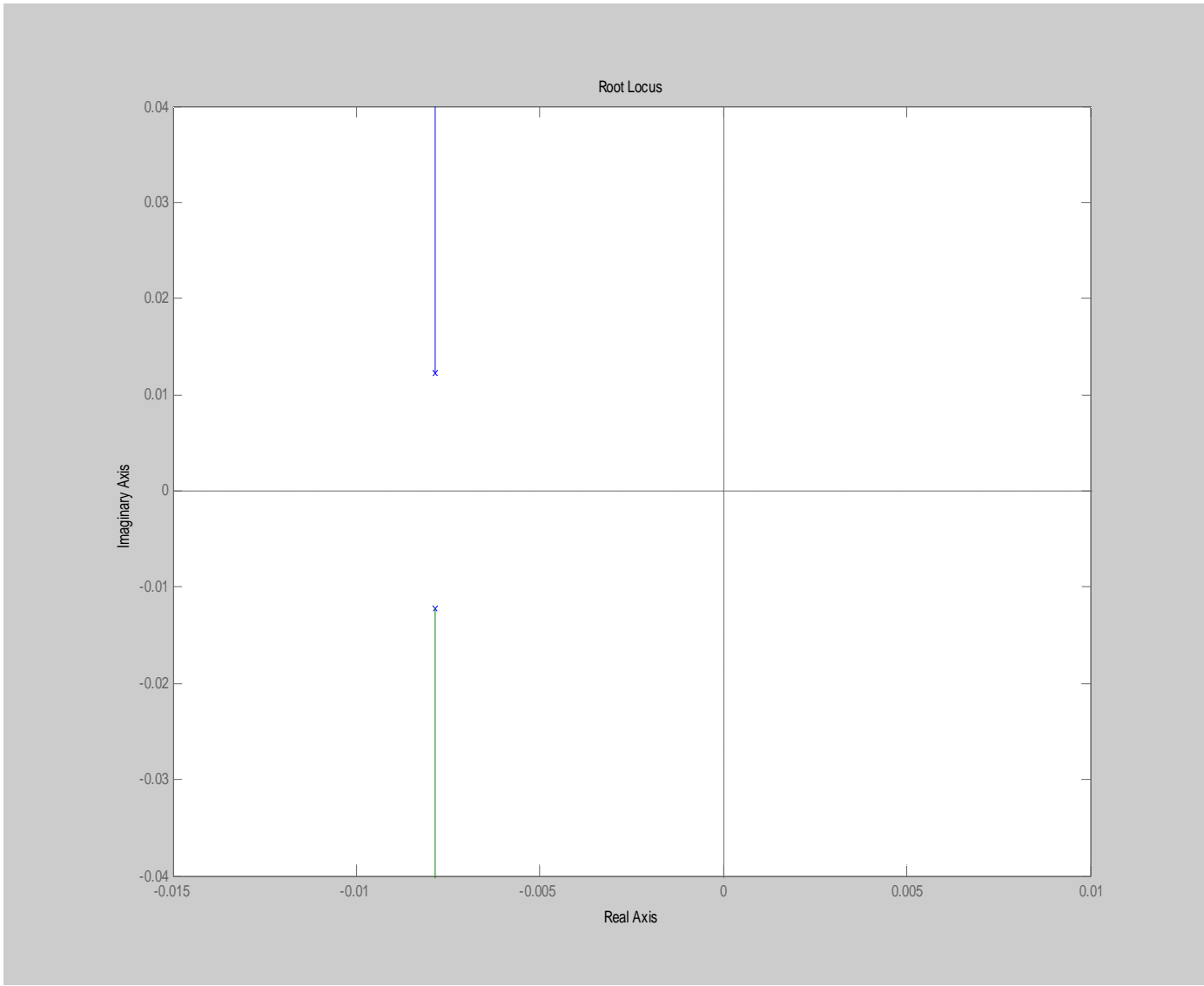
```
num = [144.07];
>> den = [49000 770 10.36];
>> G = tf(num,den)
```

Transfer function:

$$\frac{144.1}{49000 s^2 + 770 s + 10.36}$$

```
>> rlocus(G)
```





Graph no; 5A

```
>> num = [144.07];  
>> den = [49000 770 1];
```

```
>> H = tf(num,den)
```

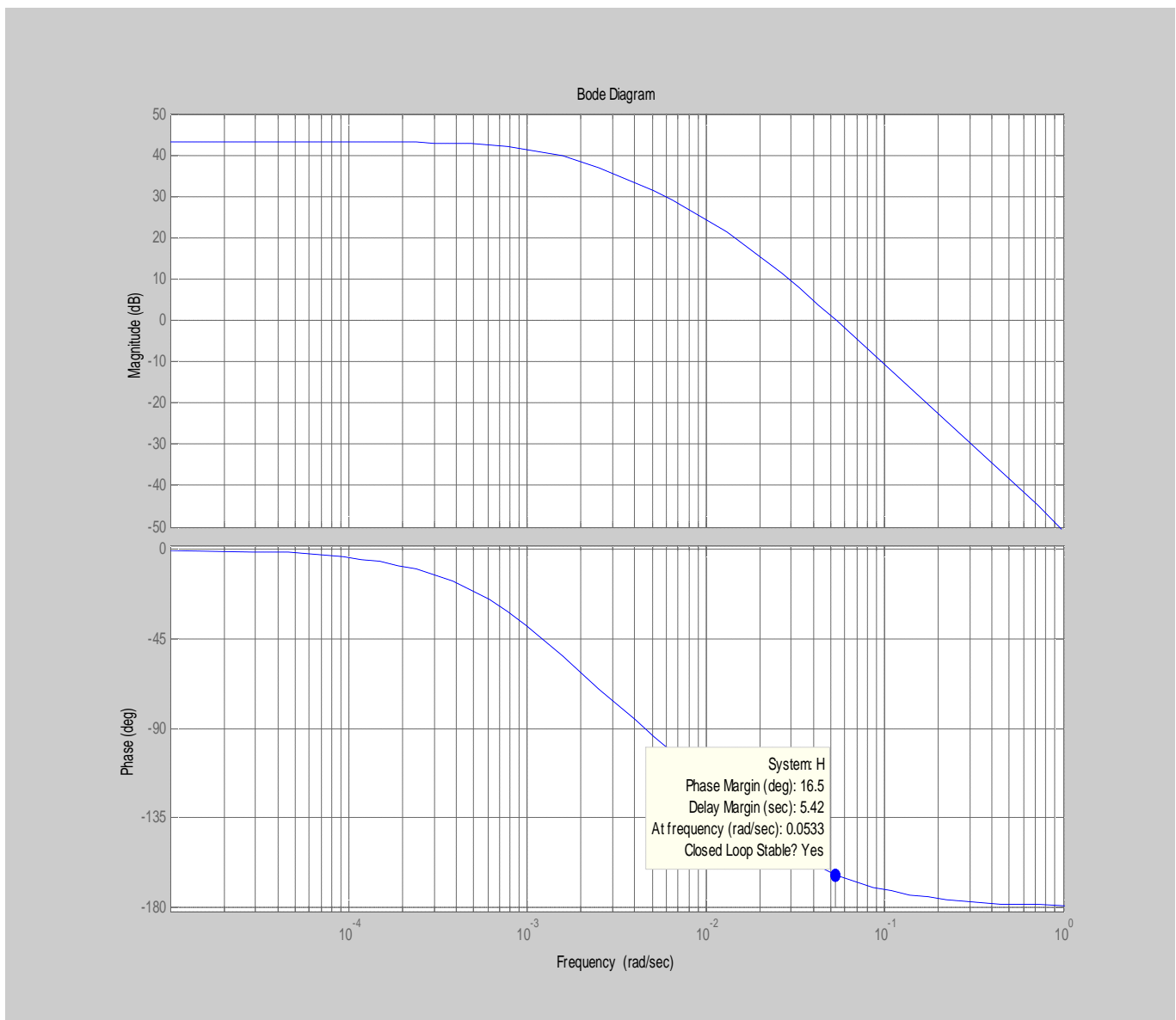
Transfer function:

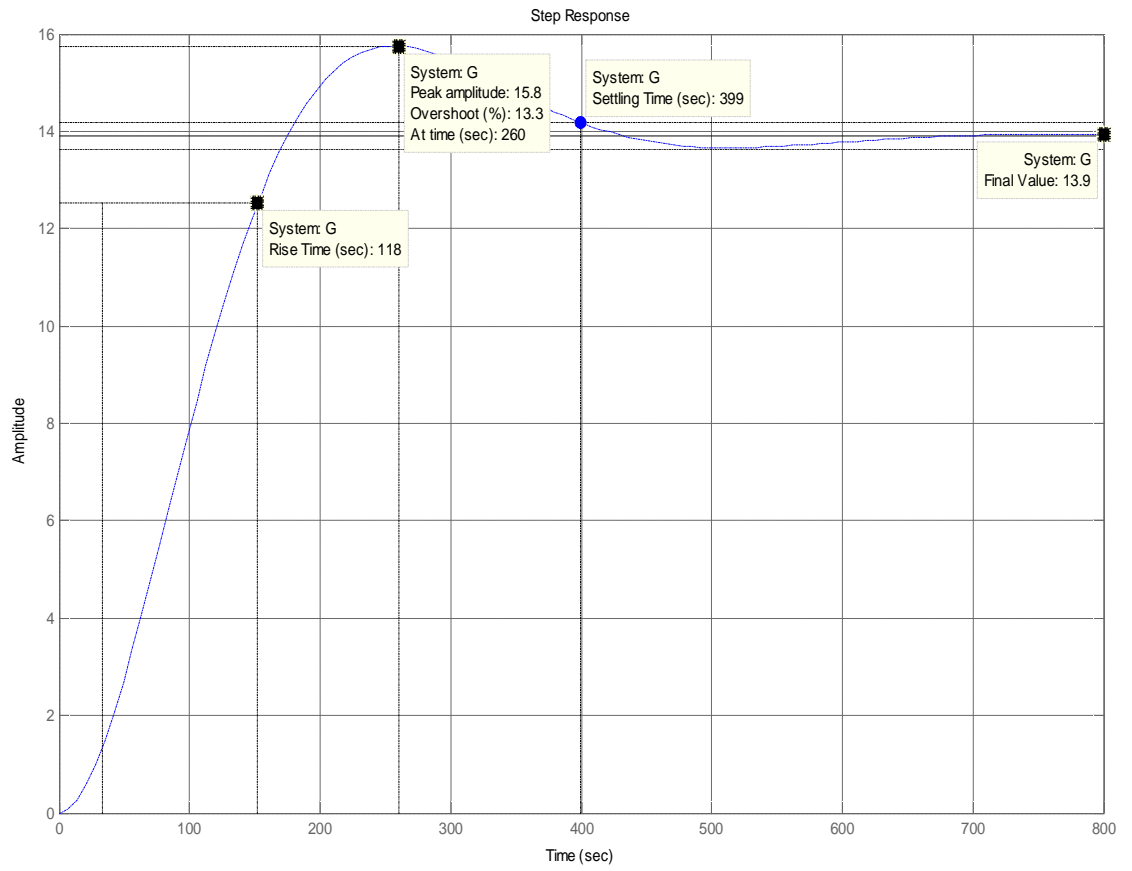
144.1

-----  
49000 s<sup>2</sup> + 770 s + 1

```
>> bode(H)
```

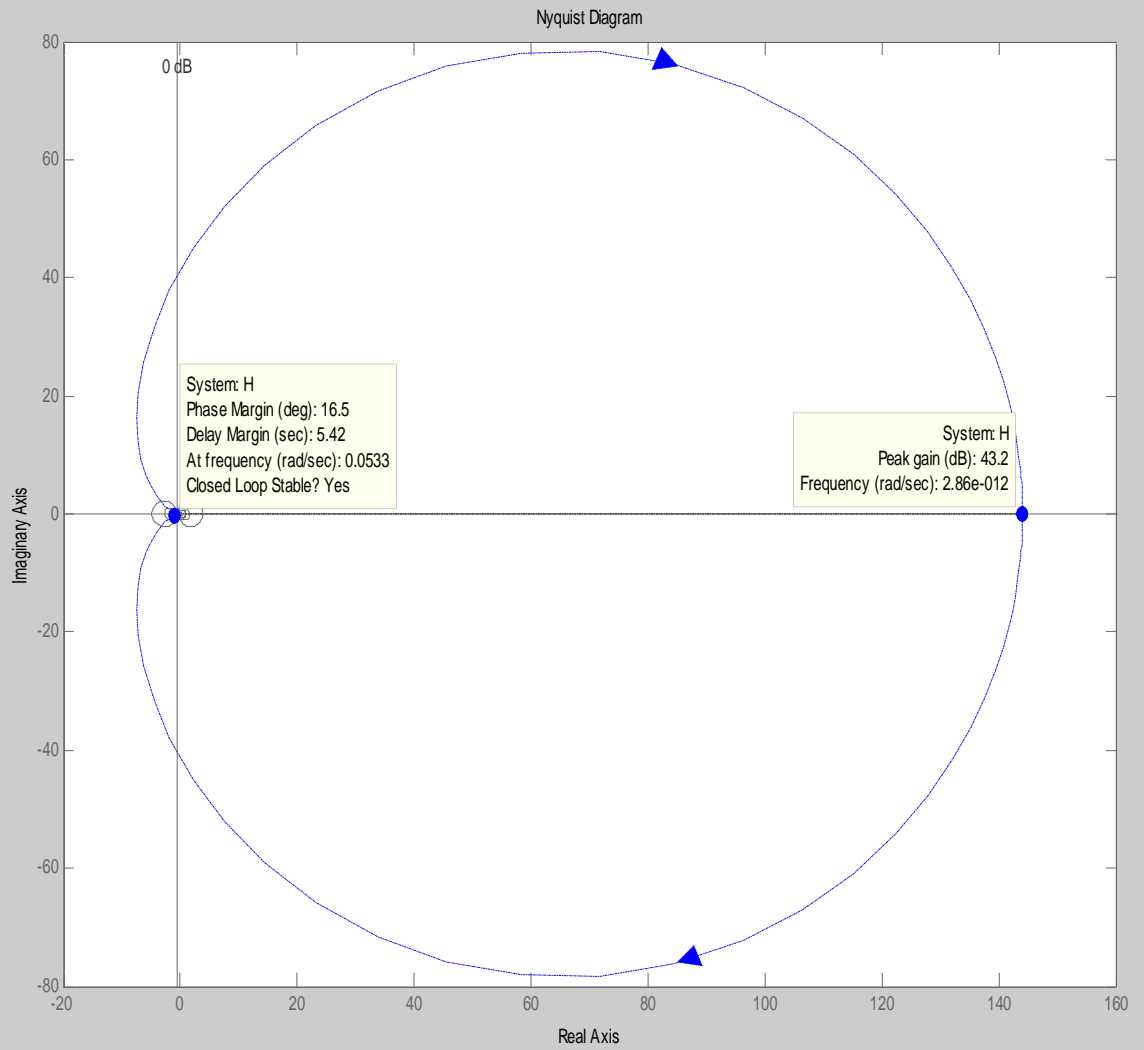
GRAPH NO 5B





nyquist(H)

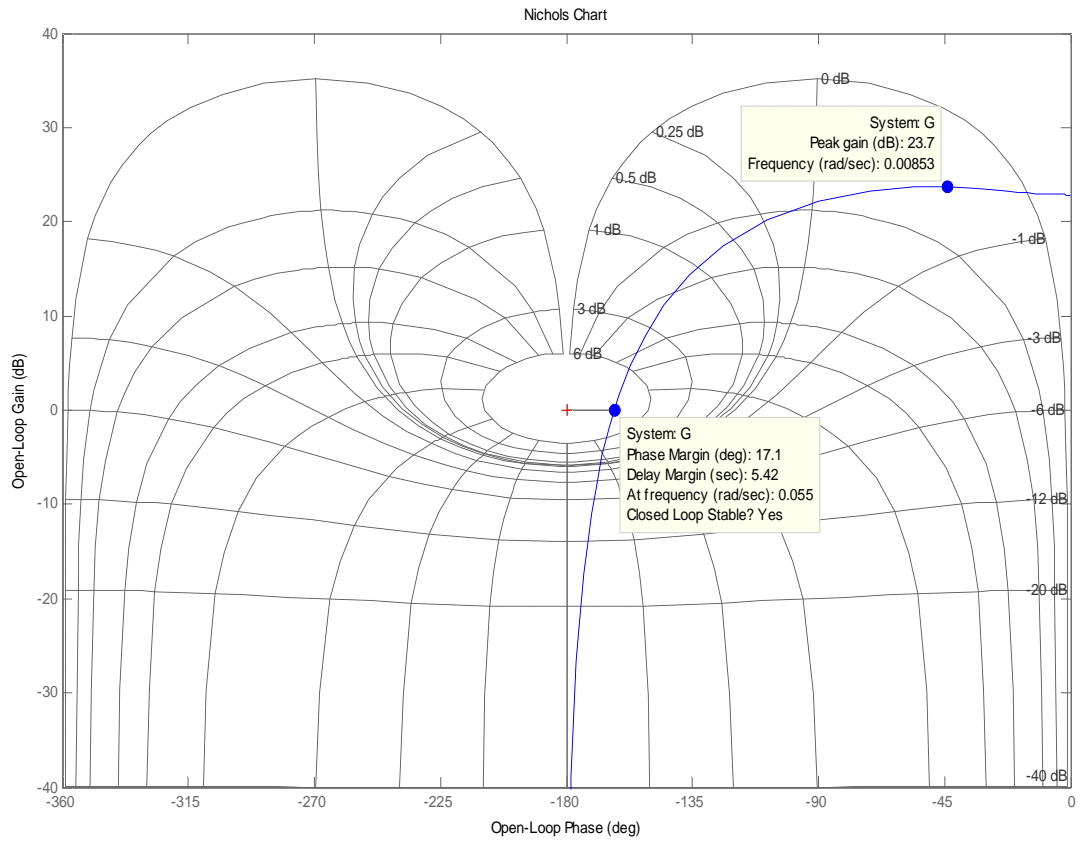
GRAPH NO 5C



GRAPH NO; 5D

>>nichols(G)

G



GRAPH NO 5E

## CHAPTER FIVE

### 5.0 CONCLUSION

For this project design there was no any instant did the system show signs of instability even with the PI controlled added on into the system. This means that although the Ward Leonard system is certainly of orders higher than two. Approximation as shown in the modeling equations in chapter three with a second order system is extremely good.

By use of the PI controller it improved the damping and reduced the maximum overshoot.

This also increased the risetime as shown in the unit step, that is graph No 5Bí that is the Bode Plot, there was improved gain margin and phase margin and hence giving Avery stable system. Due to the problem in working with the Polar coordinates at the nyquist of  $G(j\omega)$  that the curve no longer retains its original shape when a simple modification such as the change of loop gain is made to the system; the nicholes chart was plotted in graph no 5Eí and this gave a peak gain (DB) 23.7w and phase margin (deg) 17.1 and delay margin (sec) 5.42 at frequency 0.055 and thus the close loop system was very stable.

For design work involving resonant Peak Mr and Bandwidth (Bw) as specification. It was more convenient to work with the magnitude ó phase plot at  $G(j\omega)$  since when the loop gain is altered the entire  $G(j\omega)$  curve is shifted up or down vertically without distortion. Graphs No, 1,2,3,4 gave very good characteristics to determine the gradient and hence the relevant constants.

The necessary and sufficient condition that all roots of the characteristics equation as shown in chapter four lie in the left half of the s-plane is that the equations Hurwitz determination.

There was no any sign change in the first column in the Routhø Hurwitz tabulation and this shows there is an absolute stability.

The gain of 1500 rpm should have been capable of reducing drop considerably even with a load of 3kw, remembering that only a gain of 28 was required in the system on no load to give the response shown on no graph No 5C, that is the unit step response. This leaves

us with the PI controller as the sole victim, may be there was no integration at all, the system could be affected by disturbance and noise as in the case of an aircraft.

## **5.1 RECOMMENDATION**

In this work, voltage control was done using the energy wasting rheostat to provide a variable voltage. This instead could be done by the use of voltage choppers which uses chopper circuit to provide variable dc voltage from affixed dc supply. this dc supply is to be switched on off at high frequency using electronic switching devices such as MOSFETs,IGBTs,or GTOs to provide a pulsed DC wave form.

In position control of the motors potentiometers were used to provide position of the feedback in closed loop systems but shaft encoders could be used to provide more precise travel feedback by counting pulses.

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