Acknowledgement

I gratefully acknowledge the efforts of Mr. Ogaba my project supervisor, in providing me direction and guidance during the entire course of my project work. It has been pleasure and honour to have been his student. I would like to recognize and appreciate my classmates for their challenging criticism in my progress of the project and whose encouragement helped me greatly, much appreciation too goes to staff of Electronics lab for their invaluable support.
Dedication

This work is dedicated to my parents Richard and Mary Chebukto, their tireless efforts to keep me in school and for helping me nurture my talent.
Abstract

Air-core inductors are large-body low loss components that find frequent use in power electronic converters and filter circuit design. The inductance value of such an inductor depend on its geometry which can be varied by change in diameter of core, length of core, diameter of wire and number of turns. It is often necessary to construct an inductor because specific inductance values are difficult to find. In this project an investigation is carried out to determine how inductance value changes with a single test parameter variation with all others being kept constant. The main contributions of this project are a specific description of a practical approach to determine inductance, where the inductor is connected in series with a resistor and RL circuit properties are used to determine inductance value of a closely packed turn air core solenoid inductors, the experimental results demonstrate the effect on inductance value of a single parameter change at a time. The results suggest that a specific air core inductance value can be realized by a wide variety of core inductor geometries.
NOMENCLATURE

L

\text{inductance of a coil in henrys (H)}

\varphi

\text{Number of flux lines contained in the loop (webers)}

B

\text{Flux density (webers/m}^2\text{)}

I

\text{Current in the loop}

N

\text{Number of turns in the solenoid}

\mu_0

\text{Permeability of air (4\pi \times 10^{-7}H/m)}

l_c

\text{Length of solenoid}

A

\text{cross sectional area of solenoid}

Z

\text{Impedance of network}

\tau

\text{Time constant}

D

\text{core diameter}

d

\text{wire diameter}

\rho

\text{Resistivity of wire copper wire } 1.724 \times 10^{-6}\Omega/\text{cm}
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CHAPTER ONE

1.0 Introduction

An inductor is the magnetic force generated especially in a coiled wire that tries to oppose any change in electric current that flows through it. This reaction of magnetic field that tries to keep the current flow at a steady rate is called inductance. An air core inductor does not depend on a ferromagnetic material to achieve the required inductance value. A non-ferromagnetic material likes glass, wood or Bakelite can be used as the core in air core inductors.

1.1 Why air core?

Air core inductors are the only one that can achieve ideal inductive reactance behavior. They have the highest stability, the tightest tolerance, a linear inductance under dynamic signal condition, greater power handling and less distortion. They have relatively low d.c. resistance, linear a.c. resistance, better and more linear quality factor at high frequency and linear phase characteristics. They also have no magnetic core hysteretic distortion, no magnetic core saturation distortion, no magnetic core non-linear inductance and no magnetic core phase distortion. They also produce no harmonic frequency.

Without the use of high permeability core the challenges to encounter though it means the use of many turns to achieve a given inductance value thus resulting in a bulky inductor, more turns means high copper losses and low self resonance.

Inductors are used extensively in power conversion circuits. They are commonly used in input and output in ac and dc systems. The concept of inductance is derived from Faraday’s Law

1.2 chapters ahead

Chapter two deals with the theory behind the behaviour of inductors and it highlights a literature review of earlier developed relationship of inductance value on the physical parameters of the inductors.

Chapter three emphasizes on the design theory and outlines an experimental procedure used to determine inductance value of a closely packed solenoid, the effect of increasing the number of layers is also outlined. Tabulated results and snapshots of oscilloscope images are included in this chapter.
Chapter four contains discussion of results and comparison with theory. Graphs plotted using MATLAB are also contained in this chapter, a relationship is of L with test parameters is developed in this chapter.

Chapter five comprises the conclusion and future work to be done to improve on performance of air core inductors
CHAPTER TWO

2.1 Faraday’s laws

This law forms the basis of determination of inductance value of wound components. The most basic Self-inductance problem is the single turn loop inductor as shown in figure 2.0. Classical Theory dictates that the self-inductance of this loop is determined by Faraday’s Law equation 2.1.0

\[ W I \frac{dS}{dt} = L \frac{di}{dt} \]

An induced electromotive force (voltage) in any circuit is always in a direction in opposition to the current that produced it.

Consider a circuit with in which a current \( I \) is flowing. The current generates a magnetic field \( B \) which gives rise magnetic flux \( \Phi \) linking the circuit. The flux is proportional to the current \( I \). From the law of magneto -statics we have

\[ \Phi = LI \]

\( L \) is purely a geometric quantity that is, it is dependent on the shape of the circuit and the number of turns.

If the current flowing in the circuit changes by a value \( dI = Ldi \) in an interval \( dt \) then magnetic flux linking the circuit changes by an amount \( d\Phi = L\,dI \) in the same time interval. According to Faraday’s law, an emf
\[
E = -n \frac{d\phi}{dt}
\]  
(2.1.3)

is generated around the circuit. Also

\[
e\text{m}f = -L \frac{di}{dt}
\]  
(2.1.4)

Consider a solenoid of length \(l\) and cross-sectional area \(A\) and suppose the solenoid has \(N\) turns, when a current \(I\) flows in the solenoid, a uniform axial field of magnitude

\[
B = \frac{\mu_0 NI}{l}
\]  
(2.1.5)

is generated in the core of the solenoid. The magnetic flux linking a single flux of the solenoid is given by \(\Phi = BA\), thus the magnetic flux linking all flux of the solenoid is given by

\[
\Phi = NBA = \frac{\mu_0 N^2 AI}{l}
\]  
(2.1.6)

The self-inductance can thus be written as

\[
L = \frac{\mu_0 N^2}{l}
\]  
(2.1.6)

2.2 Energy stored in an inductor

When an inductor of inductance \(L\) is connected to a variable DC supply, and the supply is varied to increase current flowing in the inductor from 0 to a final value \(I\), as the current through the inductor is increased an \(e\text{m}f\) in equation 2.1.4 is generated which acts to oppose the increase in current. The work done by the voltage source in order to establish current in the inductor is given by

\[
dW = Pdt = EIdt = iL \frac{di}{dt} dt = Lidi
\]  
(2.2.1)

Where \(P = -Ei\)

(2.2.2)

is the rate at which the voltage source performs the work. Integrating equation 2.2.1, we have

\[
W = L \int_0^I idi
\]  
(2.2.3)
Giving \( W = \frac{1}{2}LI^2 \) \hspace{1cm} (2.2.4)

The energy developed by the magnetic field is generated by the current flowing through the inductor.

Writing the equation 2.2.4 as a function of \( N \) and \( l \) we obtain

\[ W = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A}{2l} \left( \frac{Bl}{\mu_0 N} \right)^2 \]  \hspace{1cm} (2.2.5)

Which reduces to

\[ W = \frac{B^2}{2\mu_0} lA \]  \hspace{1cm} (2.2.6)

The volume of field–field solenoid is \( lA \) and the magnetic density of the solenoid is

\[ w = W/(lA) \]  \hspace{1cm} (2.2.7)

The energy per unit volume in the solenoid is thus given by

\[ w = \frac{B^2}{2\mu_0} \]  \hspace{1cm} (2.2.8)

When electric and magnetic field are both present then the total energy is given by

\[ W = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \]  \hspace{1cm} (2.2.9)

2.3 Power losses in wound components

Outlined here are the various causes of inefficiencies and techniques for minimizing them. Power is lost in an air inductor through several different mechanisms:

- Resistance of the windings – copper loss
- Physical vibration and noise of the core and windings.
- Electromagnetic radiation.
- Dielectric loss in materials used to insulate the core and windings.
Only the copper loss operates even when the current in the winding is not changing. The other losses all fall to zero with frequency.

### 2.3.2 Wire winding losses

*Copper loss* or winding loss is the term used to describe the energy dissipated by resistance in the wire used to wind a coil. The behavior of an inductor at high frequencies is different from that at low frequencies, at high frequencies the skin effect and come into play causing the winding resistance to increase proportionally as $\sqrt{f}$ and the inductance to decrease slightly with increase in frequency of operation. The resistance of the winding is

$$R = \rho \frac{l}{A} \quad (2.3.0)$$

Where $l$ is the length of wire forming the turns round the solenoid. The resistivity of copper wire at room temperature is $1.724 \times 10^{-8} \Omega/cm$. The length of the wire comprising an $N$-turn winding can be expressed as

$$l = N \, (MLT) \quad (2.3.1)$$

Where MLT is the mean length per turn of the winding and is function of the core geometry.

Substituting (2.3.1) in (2.3.0) we have

$$R = \rho \frac{N(MLT)}{A} \quad (2.3.2)$$

### 2.4 Loop quality factor, $Q$

Loop quality factor is a measure of the losses in an inductive circuit. The resonant efficiency of a circuit is expressed through the dimensionless quality factor $Q$. If the losses of the inductor are large, $Q$ is low. A perfect inductor has no losses; therefore, there is no dissipation of energy within the inductor and $Q$ is infinite.

Total energy loss in a loss inductor is calculated by modeling the inductor as an equivalent lossless inductor in series with a resistor.
The quality factor is equal to the ratio of the inductive reactance to the resistive loss of the inductor. Since inductive reactance is a frequency-dependent quantity, the frequency must be specified when measuring quality factor. The formula for $Q$ is written as

$$Q = \frac{2\pi fL}{R} = \frac{wL}{R} \tag{2.4.1}$$

The $Q$ of a resonant circuit is a measure of the ratio of the energy stored in it to the energy lost during one cycle of operation. All practical inductors exhibit losses due to the resistance of the wire or absorption by materials within the magnetic field surrounding it. Inductor losses are modelled as a resistance, $R$, in series with a perfect or loss free inductance $L$ as shown in fig 2.4.1

![Figure 2.4.1](image)

The value of $R$ will be greater than that of the DC resistance of the wire due to skin effect. The above formula suggests that the $Q$ of any given inductor will increase indefinitely with frequency. This is never the case because of an effect known as self resonance.

Practical values of $Q$ range from around 10 for a high loss circuit through about 100 for a reasonable one up to around 1000 with careful design and favourable condition.

### 2.5 Inductor operating frequencies

The behaviour of inductors at low frequencies is different from its behavior at high frequencies. The construction of an inductor involves cramming a large amount of wire into a small volume, and at radio frequencies, this means that the wavelength is likely to be comparable to the length of the wire. The static magnetic conception of inductance works at low frequencies because the
length of the wire used to make the coil is much shorter than the wavelength. This means that a wave entering the coil at one terminal will emerge from the other terminal with almost exactly the same phase. Thus an instantaneous view of the magnetic field surrounding the coil will be almost identical to the field produced by a direct current; in which case, the energy stored \( LI^2/2 \) will be the same as in the DC case and the inductance can be calculated accordingly. From an electrical point of view therefore, a coil operating at low frequencies looks like a lumped inductance in series with the DC resistance of the wire.

### 2.6 complex impedances

Impedance of any given network is given by equation 2.6.1

\[ V = IZ \]  

(2.6.1)

If the network consists of a single resistor, \( Z = R \). For networks containing an inductor as shown in fig 2.6.1

![Figure 2.6.1](image)

The voltage across an inductor is given by:

\[ V = L \left( \frac{dI}{dt} \right) \]  

(2.6.1)

Also

\[ V = IZ_L \]  

(2.6.2)

Setting 2.6.1 and 2.6.2 equal:

\[ L \left( \frac{dI}{dt} \right) = IZ_L \]  

(2.6.3)

\[ \frac{dI}{dt} = \frac{(Z_L / L)}{dt} \]  

(2.6.4)

\[ \ell n I = \left( Z_{L} t / L \right) + \ell n I_o \]  

(2.6.5)
\[ I = I_0 e^{Z_L t} \]  \hspace{1cm} (2.6.6)

Now if \( Z_L \) is a real positive number, the current must increase exponentially. If \( Z_L \) is a real negative number, it must decrease exponentially toward zero. If \( Z_L \) is zero, the voltage is identically zero. To get a useful relationship let \( Z_L \) vary with time. But time varying impedance is not a useful concept for a simple inductor. A viable solution is to move out of the domain of real numbers and explore the possibility of letting \( Z_L \) be imaginary or possibly complex.

\[ L \frac{d}{dt} (I_0 e^{j\omega t}) = I_0 e^{j\omega t} Z_L \]  \hspace{1cm} (2.6.7)

\[ LI_0 j\omega e^{j\omega t} = I_0 e^{j\omega t} Z_L \]  \hspace{1cm} (2.6.8)

\[ j\omega L = Z_L \]  \hspace{1cm} (2.6.9)

Since the impedance depends on the frequency. Thus different harmonics of a periodic waveform will encounter different impedances.

It should be noted that as frequency increases, the impedance of an inductor increases

2.6.1 Argand Diagram

Used to express the relationship between current and voltage across the inductor

\[ \text{Re}^{j\theta} = R \cos \theta + j R \sin \theta \]

\[ = X + j Y \]

\[ Z = j \omega L \] plots as:

Another way to express this \( Z \) is as \( \omega L e^{j\pi/2} \). \( I = I_0 e^{j\omega t} \) plots as:
The projection of \( I_o \) on the real axis, which is the real part of \( I \), is just \( I_o \cos \omega t \). The voltage across the inductor which carries current \( I \) is \( V = IZ_L \), which is:

\[
I_o e^{j\omega t} L e^{j\pi/2} = I_o \omega L e^{j\omega t + j\pi/2}
\]

The projection of \( V \) on the real axis is:

\[
I_o \omega L \cos (\omega t + \pi/2) = I_o \omega L \left[ \cos \omega t \cos \pi/2 - \sin \omega t \sin \pi/2 \right] = -I_o \omega L \sin \omega t
\]

The current through the inductor lags the voltage across the inductor by \( 90^\circ \).

2.7 Series Impedances of RL circuit

Now consider a resistor in series with an inductor in figure 2.7.1. The same \( I \) must flow through both elements.
\[ V_b = IZ_L = Ij\omega L \]  
(2.7.1)

\[ V_a = V_b + IR = Ij\omega L + IR = I(R + j\omega L) \]  
(2.7.2)

\[ V_a = IZ_{L,R} = I_o e^{j\omega t} \sqrt{R^2 + \omega^2 L^2} e^{j\phi} , \]  
(2.7.3)

Where \( \tan \phi = \omega L/R \)

\[ Z_{L,R} = \sqrt{R^2 + \omega^2 L^2} e^{j\phi} \]  
(2.7.4)

With complex impedances, all the rules for series impedance, parallel impedance, and voltage dividers will be the same as they are with resistors, since the derivations are depended only on the equation \( V = IZ \) which is comparable to \( V = IR \) in the case of resistors.

### 2.8 Inductor D.C transient effects

Consider an inductor in series with a resistor and a source of a constant EMF. At time \( t = 0 \), the switch is closed, and current will suddenly attempt to flow. The sudden change in current will cause a counter EMF to be self-induced in the coil to oppose the change in current. This counter EMF will initially be exactly equal to the applied EMF so that no current will initially flow. As the current starts to flow, its rate of increase will decrease, and the counter EMF will likewise decrease, allowing more current to flow. The current will increase exponentially until its maximum value is reached, as determined by the series resistance in the circuit.
After a fairly long period of time has passed, the initial transient effects will have died down and a steady dc current will be flowing through the inductor. This dc current will create a constant magnetic field within the inductor. Any attempt to change quickly this constant field will be opposed by the inductor.

Assume that a coil or inductor has a constant current flowing through it and all initial transient effects have decayed. At time $t = 0$, the coil will be instantly short circuited across a resistance. The field will slowly collapse, eventually causing a current to flow until there is no more magnetic field and, correspondingly, no current is flowing. This decrease in current is exponential.

### 2.9 Time Constant ($\tau$)

It is a measure of time required for certain changes in voltages and currents in RC and RL circuits. Generally, when the elapsed time exceeds five time constants ($5\tau$) after switching has
occurred, the currents and voltages have reached their final value, which is also called steady-state response.

The time constant of an RL circuit is the equivalent inductance divided by the Thévenin resistance as viewed from the terminals of the equivalent inductor.

\[ \tau = \frac{L}{R} \]

Current in an RL circuit has the same form as voltage in an RC circuit: they both rise to their final value exponentially according to

\[ 1 - e^{t/\tau} \]

The expression for the current build-up across the Inductor is given by

\[ i(t) = \frac{V}{R} \left( 1 - e^{-\frac{R}{L}t} \right), \quad t \geq 0 \]  

(2.8.1)

Where, V is the applied source voltage to the circuit for \( t \geq 0 \). The response curve is increasing and is shown in Figure 2.8.3

![Fig. 2.8.3 Current build up across Inductor in a Series RL circuit. (Time axis is normalized by \( \tau \)](image)

The expression for the current decay across the Inductor is given by:

\[ i(t) = i_0 e^{-\frac{R}{L}t}, \quad t \geq 0 \]  

(2.8.2)

Where, \( i_0 \) is the initial current stored in the inductor at \( t = 0 \) and \( L/R = \tau \) is time constant.

The response curve is a decaying exponential and is shown in Figure 2.8.3
2.10 Air core inductor specifications

a) Inductance - The ability of air core inductor which tries to keep the current flowing through it at a steady rate. It is measured in Henries (1 Henry is the induced voltage of 1V when the current is varying at a rate of 1V/s).

b) Inductive reactance ($X_L$) is impedance provided by an inductor to current flow, it increases with frequency.

c) Inductive tolerance - Allowed variation from the nominal value

d) Direct current resistance – is the opposition of an inductor to a model direct current applied from the source.

e) Maximum dc current (IDC)- The level of continuous dc current that can be passed through an inductor without damaging it, based on a maximum temperature rise at the maximum rated transient temperature.

f) Inductance temperature coefficient- change in inductance per unit temperature rise.

g) Resistance temperature coefficient- change in dc wire resistance per unit temperature change.

h) Self resonant frequency (SRF) – the frequency at which inductor’s distributed capacitance resonates with inductance. At this frequency $L = C$ and the two effects cancel each other. As a
consequence at SRF, the inductor acts as a purely resistive high-impedance element. Also at this frequency Q= 0. Distributed capacitance is caused by the turns of core layered on top of each other and around the core.

2.11 Applications of inductors

1. Filter design

Filters are used to pass a desired range of frequencies and attenuate all other frequencies; inductors when used with other suitable electrical elements can be used in filter design.

a) Low pass filter

\[ \text{Vin} \quad R \quad V_{\text{out}} \]

b) High pass filter

\[ \text{Vin} \quad R \quad L \quad V_{\text{out}} \]

c) Band pass filter

\[ \text{Vin} \quad C \quad L \quad V_{\text{out}} \]
2. Regulator design (Buck, boost and buck and boost regulators)

3. Op amp oscillator design

4. Colpits oscillator and Hartley oscillators

5. Radio circuits – SW receiver, RF oscillator/ transmitter
CHAPTER THREE

3.0 design basis (series RL-circuit)

![Diagram of a RL circuit](image)

The relationship of $V_R$, $V_L$ and $V_S$ in a RL circuit is represented by vector diagram 3.1.

![Vector diagram](image)

Voltage drops are shown in voltage triangle OAB in figure 3.1. Vector OA represents the ohmic drop $V_R$ and AB represents inductive $V_L$. The applied voltage is the sum of the two i.e.

$$V_S = \sqrt{(V_R^2 + V_L^2)} = \sqrt{(IR)^2 + (I.X_L)^2} = \sqrt{IR^2 + X_L^2},$$

$$I = \frac{V_S}{\sqrt{R^2 + X_L^2}}$$
\[
\tan \phi = \frac{V_L}{V_R} = \frac{I_X}{\frac{1}{R}} = \frac{X_L}{R}
\]

The resistance was chosen to have a value almost equal to the reactance of the inductor so that the \( V_R \sim V_L \), with this consideration then \( I = \frac{V_R}{R} \).

The inductance value was calculated by we use the equation 3.0.1 obtained from: \( V_L = I \cdot X_L = I \cdot 2\pi f L \). Where \( f \) is set from the signal generator.

\[
L = \frac{V_L}{2\pi f}
\]  

(3.0.1)

3.1 Investigating effect of inductance on parameter change

The diameter of wires were measured using a micrometer screw gauge was converted to a corresponding SWG value using table B in appendix.

3.1.0 Effect of varying the diameter of the core on inductance

The diameters of 6 separate wooden cores were measured by vernier callipers and were found to be 10.8mm, 12.7mm, 16.4mm, 20.6mm, 29.9mm and 44.4mm.

With \( d, N \) and \( l \) kept constant the results obtained from the measurements were recorded in table 3.3.4. \( L \) was calculated from the figures in the table.

3.1.1 Effect of varying the number of turns on inductance value

With \( D \) and \( d \) kept constant 77,124 and 160 turns were made on the circular cores. Turns of 237,284 and 361 were made by connecting suitable solenoids in series. Results obtained and the inductances value calculated were recorded in table 3.3.1.
3.1.2 Effect of varying the length of core on inductance value

Cores of lengths 54mm, 92mm, 116mm, 146mm, 170mm, 208mm and 262mm were used to investigate this effect. D, N and d were kept constant. The calculated inductance as a result of varying core length was recorded in table 3.3.2

3.1.3 Effect of varying the diameter of wire on the inductance value

Wires of diameters 0.33, 0.46, 0.49 and 0.69 were wound on core of diameter 6.23mm and length 28mm. The turns were tightly packed; inductance value was calculated and recorded in table 3.3.3

3.1.4 Effect of varying the number of layers of wire

Solenoids of lengths and layers tabulated in 3.3.5 were tested and their inductance value calculated. Their inductance value was calculated and recorded.

3.2 Laboratory procedure to obtain inductance of a solenoid

The objective of this laboratory procedure was to investigate the inductance concept and observe the voltage drop across the inductor.

3.2.0 Equipment:

• Digital Multi-meter
• Signal generator
• Oscilloscope
• Resistors
• Solenoids of different parameters
3.2.1 Procedure

1. The internal resistance of the coil (r) was measured using a multi-meter and its value recorded. The signal generator consisted of a variable emf source ($V_s$) and variable frequency source.

2. The apparatus were connected as indicated in fig 3.2.1 the negative side of the function generator was grounded. Channel 1 of the oscilloscope connected across the source and channel 2 connected across the R.

3. Resistors R were chosen with a value close to the reactance of the inductor, this was achieved by choosing a resistor which gave a voltage drop with amplitude approximately half that of input voltage. Where need be resistors were connected in parallel or series to give the desired value of resistance.

4. The current in the circuit was calculated by, $I = \frac{V_R}{R}$

5. The procedure was repeated for all the coils and their values recorded in table’s 3.3.1-3.3.5

6. The inductance of each test coil was calculated by equation 3.0.1
### 3.3 Results

Table 3.3.1 showing how inductance varies with the number of turns

<table>
<thead>
<tr>
<th>N</th>
<th>D (mm)</th>
<th>$V_s$ (mv)</th>
<th>$V_R$ (mv)</th>
<th>$V_I$ (mv)</th>
<th>R(Ω)</th>
<th>I (mA)</th>
<th>L (μH)</th>
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</thead>
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<td>100</td>
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<td>141</td>
<td>23.3</td>
<td>6.1</td>
<td>36.8</td>
</tr>
<tr>
<td>284</td>
<td>6</td>
<td>200</td>
<td>140.23</td>
<td>142.6</td>
<td>30</td>
<td>4.67</td>
<td>46</td>
</tr>
<tr>
<td>361</td>
<td>6</td>
<td>200</td>
<td>139.4</td>
<td>143.4</td>
<td>50</td>
<td>2.8</td>
<td>81.5</td>
</tr>
</tbody>
</table>

Table 3.3.2 showing how inductance varies with length of core

<table>
<thead>
<tr>
<th>l (mm)</th>
<th>D (mm)</th>
<th>$V_s$ (mv)</th>
<th>$V_R$ (mv)</th>
<th>$V_I$ (mv)</th>
<th>R(Ω)</th>
<th>I (mA)</th>
<th>L (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>6</td>
<td>100</td>
<td>68.34</td>
<td>73</td>
<td>2.5</td>
<td>27.3</td>
<td>4.3</td>
</tr>
<tr>
<td>92</td>
<td>6</td>
<td>200</td>
<td>145.7</td>
<td>137</td>
<td>6.67</td>
<td>21.84</td>
<td>9.98</td>
</tr>
<tr>
<td>116</td>
<td>6</td>
<td>200</td>
<td>144.3</td>
<td>138.5</td>
<td>10</td>
<td>14.43</td>
<td>15.3</td>
</tr>
<tr>
<td>146</td>
<td>6</td>
<td>200</td>
<td>143</td>
<td>139.7</td>
<td>16.7</td>
<td>8.6</td>
<td>25.87</td>
</tr>
<tr>
<td>170</td>
<td>6</td>
<td>200</td>
<td>141.8</td>
<td>141</td>
<td>23.3</td>
<td>6.1</td>
<td>36.8</td>
</tr>
<tr>
<td>208</td>
<td>6</td>
<td>200</td>
<td>140.23</td>
<td>142.6</td>
<td>30</td>
<td>4.67</td>
<td>46</td>
</tr>
<tr>
<td>262</td>
<td>6</td>
<td>200</td>
<td>139.4</td>
<td>143.4</td>
<td>50</td>
<td>2.8</td>
<td>81.5</td>
</tr>
</tbody>
</table>

Table 3.3.3 showing effect of varying diameter of wire on inductance

<table>
<thead>
<tr>
<th>d (mm)</th>
<th>SWG</th>
<th>$V_s$ (mv)</th>
<th>$V_R$ (mv)</th>
<th>$V_I$ (mv)</th>
<th>R(Ω)</th>
<th>I (mA)</th>
<th>L (μH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.33</td>
<td>29</td>
<td>150</td>
<td>105.1</td>
<td>107</td>
<td>10</td>
<td>10.5</td>
<td>16.2</td>
</tr>
<tr>
<td>0.46</td>
<td>26</td>
<td>150</td>
<td>105.9</td>
<td>106.2</td>
<td>6.7</td>
<td>15.9</td>
<td>10.6</td>
</tr>
<tr>
<td>0.49</td>
<td>25</td>
<td>150</td>
<td>106.9</td>
<td>105.4</td>
<td>5</td>
<td>7.8</td>
<td>7.8</td>
</tr>
<tr>
<td>0.69</td>
<td>22</td>
<td>150</td>
<td>107.2</td>
<td>104.9</td>
<td>3.3</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 3.3.4 showing variation of inductance value with core diameter change

<table>
<thead>
<tr>
<th>N</th>
<th>$D$(mm)</th>
<th>$V_\text{s}(mv)$</th>
<th>$V_\text{R}(mv)$</th>
<th>$V_\text{L}(mv)$</th>
<th>R($\Omega$)</th>
<th>I(mA)</th>
<th>L($\mu$H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>10.8</td>
<td>150</td>
<td>114</td>
<td>98</td>
<td>40</td>
<td>2.85</td>
<td>5</td>
</tr>
<tr>
<td>31</td>
<td>12.7</td>
<td>150</td>
<td>112</td>
<td>99.8</td>
<td>40</td>
<td>2.8</td>
<td>6.6</td>
</tr>
<tr>
<td>31</td>
<td>16.4</td>
<td>150</td>
<td>109</td>
<td>103</td>
<td>60</td>
<td>1.82</td>
<td>9</td>
</tr>
<tr>
<td>31</td>
<td>20.6</td>
<td>150</td>
<td>107</td>
<td>105</td>
<td>100</td>
<td>1.07</td>
<td>15.4</td>
</tr>
<tr>
<td>31</td>
<td>29.9</td>
<td>150</td>
<td>104</td>
<td>108</td>
<td>165</td>
<td>0.69</td>
<td>27.2</td>
</tr>
<tr>
<td>31</td>
<td>44.4</td>
<td>150</td>
<td>101</td>
<td>110</td>
<td>300</td>
<td>0.34</td>
<td>52.6</td>
</tr>
</tbody>
</table>

Table 3.3.5 Effect of increasing the number of layers on inductance

<table>
<thead>
<tr>
<th>layers</th>
<th>N</th>
<th>$l$(mm)</th>
<th>$V_\text{s}(mv)$</th>
<th>$V_\text{R}(mv)$</th>
<th>$V_\text{L}(mv)$</th>
<th>L($\mu$H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>384</td>
<td>206</td>
<td>170</td>
<td>54</td>
<td>113</td>
<td>92.79</td>
</tr>
<tr>
<td>2</td>
<td>383</td>
<td>90</td>
<td>215</td>
<td>115</td>
<td>98</td>
<td>227.85</td>
</tr>
<tr>
<td>3</td>
<td>383</td>
<td>61</td>
<td>270</td>
<td>175</td>
<td>95</td>
<td>357.7</td>
</tr>
<tr>
<td>4</td>
<td>383</td>
<td>46</td>
<td>320</td>
<td>230</td>
<td>87</td>
<td>513.32</td>
</tr>
</tbody>
</table>
Figure 3.3.6 the bigger magnitude waveform indicates voltage from the supply and the smaller amplitude indicates voltage across the resistor, the vector difference indicates the voltage across the inductor.

Fig 3.3.6 showing $V_S$ and $V_L$ snapshot taken in an oscilloscope.
CHAPTER FOUR

4.0 Results analysis and comparison with theory

To aid analysis; graphs showing variation of inductance with number of turns, variation of inductance with diameter of wire, variation of inductance with diameter of core and variation of inductance with length of core were plotted on different scales using MATLAB. A line of best fit was plotted for each graph, this is done to take care of experimental errors in the lab and to cater for truncation in calculus brought about by calculation of the inductance value, other errors which may have brought about imperfect curves were the reading errors of measuring instruments used (micrometer screw gauge, vernier calipers and ruler). The graphs plotted are labelled 4.0.1-5

4.0.1 Graphs

figure 4.0.1

![Graph showing variation of inductance with change in number of turns](image)
Figure 4.0.2

VARIATION OF INDUCTANCE WITH CHANGE IN DIAMETER OF WIRE

Figure 4.0.3

VARIATION OF INDUCTANCE WITH CHANGE IN DIAMETER OF CORE
Figure 4.0.4

VARIATION OF INDUCTANCE WITH CORE LENGTH

![Graph showing variation of inductance with core length.]

Figure 4.0.5

EFFECTS OF INCREASING NUMBER LAYERS ON INDUCTANCE

![Graph showing effects of increasing number layers on inductance.]

26
With curve of best fit tool in MATLAB the equation for curve of best fit in each graphs are

**4.0.2 Discussion and comparison with theory.**

From figure 4.0.1 the best fit approximation of curve is a quadratic function with relation

\[ L = 6.48 \times 10^{-4} N^2 - 0.0165N + 0.05 \tag{4.0.4} \]

From the relation a long straight current carrying conductor which can be seen to be composed of a single turn possesses small values of inductance, for large \( N^2 \gg N \) that is \( L \propto N^2 \) because other terms are negligible.

The more the number of turns of wire means that the coil will generate a greater amount of magnetic field force, for a given amount of coil current. Each turn forms a loop round the conductor; the flux field produced by one loop cuts the other loops consequently as the number of turns is increased keeping the length and the diameter of the core constant, the flux produced by one loop will cut many other loops hence greater value of inductance is obtained. Doubling the number of turns in the coil will produce a field twice as strong, if the same current is used. A field twice as strong, cutting twice the number of turns will induce four times the voltage. Therefore it can be said that the inductance varies as the square of the number of turns \( L^2 \propto N^2 \) as per analysis.

In figure 4.0.2 a wire with smaller diameter has higher inductance value because many turns can be packed over the same length of core and since \( L \propto N^2 \), a decrease in wire diameter has the effect of increasing inductance. The relation of \( L \) with \( d \) is

\[ L = -30d + 25 \tag{4.0.5} \]

Diameter of the wire affects the span of length of the wire covered by the winding. For a single layer coil number of turns \( \times \) diameter of the wire should not exceed the length of the core. For small \( d \) many turns can be made over the same length of core and since \( L \) has a quadratic relation with \( N \), inductance decrease with decrease in diameter. This value approximates relation up to
d=0.833 mm (SWG=21). To expand this relation to include bigger diameters, a variety of large size wire diameters need to be tested.

From the figure 4.0.3 the curve of L against D the line of best fit is a quadratic function in the, MATLAB gives the equation of the function as

\[
L = 0.018D^2 + 0.478
\]  

(4.0.6)

This relation indicates that for very small diameters the inductance is \( \approx 0.478 \), and for reasonably large diameters \( L \propto D^2 \). Accuracy of this relation improves with increase in diameter.

A coil with a larger diameter means physically the inductor requires more length of wire to construct than a core of a smaller diameter with equal number of turns. Greater coil area presents less opposition to the formation of magnetic field flux, for a given amount of field force therefore more lines of force exist to induce counter emf in the coil with larger diameter.

Actually, the inductance of the coil increases as the cross-sectional area of the core increases.

Figure 4.0.4 which gives the relation of inductance with length of core, since the length of wire is not a constant and that turns are tightly packed a longer core has more turns than a shorter one and since and consequently more flux linkage in the longer core. From theoretical equation given in 2.1.6 presented in chapter two, wire length was a constant and a longer core means wider spacing between consecutive turns which increases flux leakage (reduce flux linkage), that led to the inverse relationship of inductance with core length, here we assume closely packed turns and the wire used is not of unlimited supply. The quadratic curve has a relation.

\[
L = 1.2 \times 10^{-3} l_c^2 - 0.022 l_c + 1.98
\]

For large \( l \),

\[
L = 1.2 \times 10^{-3} l_c^2 - 0.022 l \]  

(4.0.7)

Combining equation 4.0.4, 4.0.5, 4.0.6 and 4.0.7
We obtain

\[ L^4 = (6.48 \times 10^{-4} N^2 - 0.0165N+1.9731)(-30d+25)(0.018D^2+0.478)(1.2 \times 10^{-3} l_c^2 - 0.022l_c) \]

\[ L \sim 4 \sqrt{6.48 \times 10^{-4} N^2(-30d + 25)(0.018D^2)(1.2 \times 10^{-3} l_c^2 - 0.022l_c)} \]

This relates the inductance with test parameters for single layer air core coil inductor.

The inductance value can be increased by a great factor by winding the turns in layers as shown in table 3.3.5

Where \( L \) is in µH.

According to theory there are developed formulas which give inductors of air coil inductors, for single layer Wheeler’s formula shown in equation 4.0.7 best approximates the inductance value

\[ L = \frac{10 \pi \mu_0 N^2 a^2}{9a+10l} \quad \text{where } a=\text{radius of solenoid} \]

For multilayer the formula does not hold because the incidence of turn to turn, turn to layer magnetic coupling is increased

The formula which is used is

\[ L = \frac{0.8r^2 N^2}{6r+9l+10d} \]

Where \( d = \text{depth of winding (outer diameter- inner diameter)} \)

\( r = \text{mean radius of coil} \)

\( l = \text{length of coil in inches} \).
The formulas does not put into consideration the wire gauge the reason could be the aspect wire
gauge and length of core are related by \( d = \frac{L_c}{N} \) once the length of core and \( N \) are considered then
by intuition the wire gauge effect comes into play.

4.0.3 Number of turns for typical crossover inductors

Typical crossover used inductors are valued at \( L_1 = 2.82 \text{mH} \) and \( L_2 = 0.42 \text{mH} \)

This value is high to be achieved by single layer air core inductor therefore we use multilayer air
core.

Possible multi-layer core to achieve the above results is obtained by extrapolating results of
figure 4.0.5.

The number of turns required to obtain an inductance value of \( 0.42 \text{mH} = 420 \mu \text{H} \) is a 3 layer
\( N=383 \), core length is 70mm, \( d=0.46 \text{mm} \) and \( D = 13 \text{mm} \)

The number of turns required to obtain an inductance value of \( 2.82 \text{mH} = 2.82 \times 10^3 \mu \text{H} \) is a 6
layer=383, core length is 70mm, \( d=0.46 \text{mm} \) \( D = 13 \text{mm} \).
CHAPTER FIVE

5.0 Conclusion and further work

A specific air core inductance value can be obtained by changing the various geometric parameters of the inductor. Generally the inductance value of air core inductors is low but because of their linear behavior and lack of harmonics they find wide use in high frequency requirement applications. An ideal inductor described as one with no internal resistance and does not dissipate or radiate energy is not achievable in practice because inductors are made of wires. Inductors do not behave like resistors which simply oppose current flow inductors oppose changes in current through them; they drop a voltage directly proportional to the rate of change of current, this induced voltage is always of such a polarity as to try to maintain current at its present value. The voltage drop across the inductor is a reaction to change in current across it. Because instantaneous power is the product of the instantaneous voltage and the instantaneous current the power equals zero whenever the instantaneous current or voltage is zero. Whenever the instantaneous current and voltage are both positive, the power is positive. In inductor because the current and voltage waves are 90° out of phase, there are times when one is positive while the other is negative, resulting in equally frequent occurrences of negative instantaneous power. Negative power means that the inductor is releasing power back to the circuit, while a positive power means that it is absorbing power from the circuit. Since the positive and negative power cycles are equal in magnitude and duration over time, the inductor releases just as much power back to the circuit as it absorbs over the span of a complete cycle. The inductive opposition to alternating current is similar to resistance, but different in that it always results in a phase shift between current and voltage, and it dissipates zero power.

If the frequency equals zero, then so does the impedance - a frequency of zero means DC, so inductors have virtually no resistance to DC current flow but does not allow whole of source current to flow through it because it possesses internal resistance. And as the frequency goes up, so does the impedance. This is opposite behaviour compared to capacitance. Future work in the air core inductor field includes:
➢ To develop computer software that gives the inductance when given inductor specification.
➢ Design a RCL bridge that measures inductance value
➢ To investigate the dimensions which give optimum inductance per unit area.
➢ Investigate on possibility of developing an inductor which comes as an integrated chip
APPENDIX A

1 Inductor networks

Inductors in series

\[ L_T = L_1 + L_2 + \cdots + L_n \]

Inductors in parallel

\[ \frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \]

a) Inductor Response

Natural (homogenous) response of an inductor

\[ V(t) = L \frac{di}{dt} \]

\[ I(t) = \frac{1}{L} \int_0^t V(t') dt' + I(0) \]

Natural Response:

\[ V(t) = V_0 e^{-t/\tau} \]

\[ I(t)_{\text{Inductor}} = -\frac{V_0}{R} e^{-t/\tau} \]

Step Response

\[ I(t) = \frac{V_s}{R} \left( 1 - e^{-t/\tau} \right) + I_0 e^{-t/\tau} \]

\[ V(t) = V_s e^{-t/\tau} - R I_0 e^{-t/\tau} \]
2 KCL Node Voltage Method

Writing the KCL at V(t), current leaving the node is positive,

\[
\frac{V}{R} + \frac{1}{L} \int_0^t V(t') dt' + I(0) = 0
\]

Differentiate once

\[
\frac{1}{R} \frac{dV}{dt} + \frac{1}{L} V(t) = 0
\]

Solving using separation of variables:

\[
V(t) = V_0 e^{-t/\tau}
\]

where

\[
\tau = \frac{L}{R}
\]

Now that the voltage is known, the current through the resistor is just \(V/R\), or

\[
I(t)_{\text{Resistor}} = \frac{V_0}{R} e^{-t/\tau}
\]

Find the current through the inductor by integrating the voltage across the inductor:

\[
I(t) = \frac{1}{L} \int_0^t V_0 e^{-t/\tau} + I(0)_{\text{Inductor}}
\]

integrate and evaluate:
\[ I(t) = \frac{1}{L} \left( -\frac{V_0}{R} e^{-\frac{t}{\tau}} \right) + I(0)_\text{inductor} \]

\[ I(t) = -\frac{V_0}{R} e^{-\frac{t}{\tau}} + \frac{V_0}{R} + I(0)_\text{inductor} \]

But \( \frac{V_0}{R} \) is the initial current through the resistor and is equal to the negative of the current through the inductor,

\[ \frac{V_0}{R} = -I(0)_\text{inductor} \]

Thus, the final current through the inductor is

\[ I(t)_\text{inductor} = -\frac{V_0}{R} e^{-\frac{t}{\tau}} \]

or

\[ I(t)_\text{inductor} = I(0) e^{-\frac{t}{\tau}} \]

The current through the inductor is opposite the current through the resistor, per mathematical convention of the KCL.

3. Mesh loop analysis

Taking a current loop I in the clockwise direction, the KVL around the loop gives:

\[ IR + L \frac{dI}{dt} = 0 \]

solving by integration
\[ I(t) = I_0 e^{-\frac{t}{\tau}} \]

Where \( I_0 \) is the initial current through the inductor. Calculating \( V \)

\[ V = L \frac{dI}{dt} \text{ or } V = -RI_0 e^{-\frac{t}{\tau}} \]

So if the initial current \( I_0 \) is clockwise, the voltage at point \( V \) with respect to the other node is negative. If the initial current is counter clockwise, the voltage at \( V \) with respect to the other node is positive.

4. Finding the solution by direct substitution

The solution to the natural response is of the form \( K_2 e^s \).

Apparently there is some type of connection between \( d/dt \) and the exponential function.

That connection is seen in the eigenvalue equation, where the linear operator \( \hat{D} \) operates on the eigenvector \( f(t) \) to produce the eigenvalue equation,

\[ \hat{D} f(t) \] = \( \lambda \) \( f(t) \)

This equation says that the function \( f(t) \) is scaled by the operator \( D \) but is otherwise returned intact. The eigenvectors of \( \frac{d}{dt} \) are \( K_2 e^s \), where \( K_2 \) is an unknown constant, as in,

\[ \frac{d}{dt} K_2 e^s = sK_2 e^s \]

We can use this to solve

\[ \text{(1)} \quad IR + L \frac{dI}{dt} = 0 \]

we can write it

\[ \hat{D} \Phi(t) = \lambda f(t) = 0 \]

Where
\[ \dot{\mathbf{D}} = R + L \frac{d}{dt} \]

Solving for \( \lambda = 0 \) from the eigen vector.

(2) \( \text{let } I(t) = f(t) = K_2 e^s \)

Substituting (2) into (1)

\[ R K_2 e^s + s L K_2 e^s = 0 \]

Divide by \( R K_2 e^s \)

\[
-\frac{R}{L} = s
\]

Solving for the constant requires an initial condition of the circuit. At \( t=0 \) we require \( I=I_0 \), so \( K_2 \) is trivially \( I_0 \).

\[ I(t) = I_0 e^s \]

**Step Response of the Inductor / Resistor**

1. **Mesh loop analysis**

For mesh analysis,

\[ \text{the KVL equation is} \]
\[-Vs + IR + L \frac{dI}{dt} = 0\]

Solving by substitution and separation of variables gives the current in the inductor,

\[I(t) = \frac{Vs}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_0 e^{-\frac{t}{\tau}}\]

Multiply by L and take the derivative for the voltage across the inductor,

\[V(t) = Vs \ e^{-\frac{t}{\tau}} - R I_0 e^{-\frac{t}{\tau}}\]

2. KCL

This method requires special attention to the signs.

Writing KCL at V(t) gives

\[\frac{V(t) - Vs}{R} + \frac{1}{L} \int_0^t V dt + I_0 = 0\]

Multiplying by R and differentiating gives

\[\frac{dV}{dt} + \frac{R}{L} V = 0\]

Solving
\[ V(t) = V_0 e^{-\frac{t}{\tau}} \]

\( V_0 \) is \( V_s \) plus the voltage across the resistor,

\[ V_0 = V_s + I(0)_{\text{Resistor}} R \]

t when \( I_0 \) is defined counter clockwise through the loop. This is different than the \( I_0 \) solved for in \( I(t) \) by a sign.

\[ I(0)_{\text{Inductor}} = -I(0)_{\text{Resistor}} \]

so,

\[ V(t) = (V_s - RI_0) e^{-\frac{t}{\tau}} \]

as before.

To find the current in the inductor,

\[ I(t) = \frac{1}{L} \int_0^t V_0 e^{-\frac{t}{\tau}} + I(0) \]

where \( I_0 \) is defined through the inductor (opposite to the current in the resistor)

Doing the integration,

\[ I(t) = \frac{1}{L} \left[ -\frac{L}{R} V_0 e^{-\frac{t}{\tau}} \right]_{t=0}^{t} + I(0)_{\text{Inductor}} \]

\[ I(t) = -\frac{V_0}{R} e^{-\frac{t}{\tau}} + \frac{V_0}{R} + I(0)_{\text{Inductor}} \]

Remember,

\[ V_0 = V_s - R I(0)_{\text{Inductor}} \]

So for the inductor current,
\[ I(t) = -\frac{V_s - R I(0)_{\text{Inductor}}}{R} e^{\frac{-t}{\tau}} + \frac{V_s - R I(0)_{\text{Inductor}}}{R} + I(0)_{\text{Inductor}} \]

simplify,

\[ I(t) = \frac{V_s}{R} \left( 1 - e^{\frac{-t}{\tau}} \right) + I_0 e^{\frac{-t}{\tau}} \]

3. Direct Substitution method

The driven response is also called the steady state or the particular solution. It is the final value as \( t \to \infty \)

To solve

\[ (3) \quad -V_s + IR + L \frac{dI}{dt} = 0 \]

assume \( I(t) = K_1 + K_2 e^{st} \). \( K_1 \) will carry the steady state response, the second term is the natural response.

Substituting this assumption for \( I(t) \) into (3) and equating exponentials and constants each to 0 gives,

\[ s = \frac{-R}{L} \]

and

\[ K_1 = \frac{V_s}{R} \]

Thus \( I(t) = \frac{V_s}{R} + K_2 e^{\frac{-t}{\tau}} \)

At \( t=0 \) the current in the inductor is \( I_0 \), so \( K_2 = I_0 - \frac{V_s}{R} \)

giving
\[ I(t) = \frac{V_s}{R} + \left( I_0 - \frac{V_s}{R} \right) e^{-\frac{t}{\tau}} \]

or

\[ I(t) = \frac{V_s}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) + I_0 e^{-\frac{t}{\tau}} \]
### APPENDIX B

<table>
<thead>
<tr>
<th>AWG no</th>
<th>Wire diam(mm)</th>
<th>Area (mm²)</th>
<th>AWG no</th>
<th>Wire diam(mm)</th>
<th>Area (mm²)</th>
</tr>
</thead>
<tbody>
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<td>8.25</td>
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<td>19</td>
<td>0.912</td>
<td>0.653</td>
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<tr>
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*AWG/ Metric conversion table*
References


